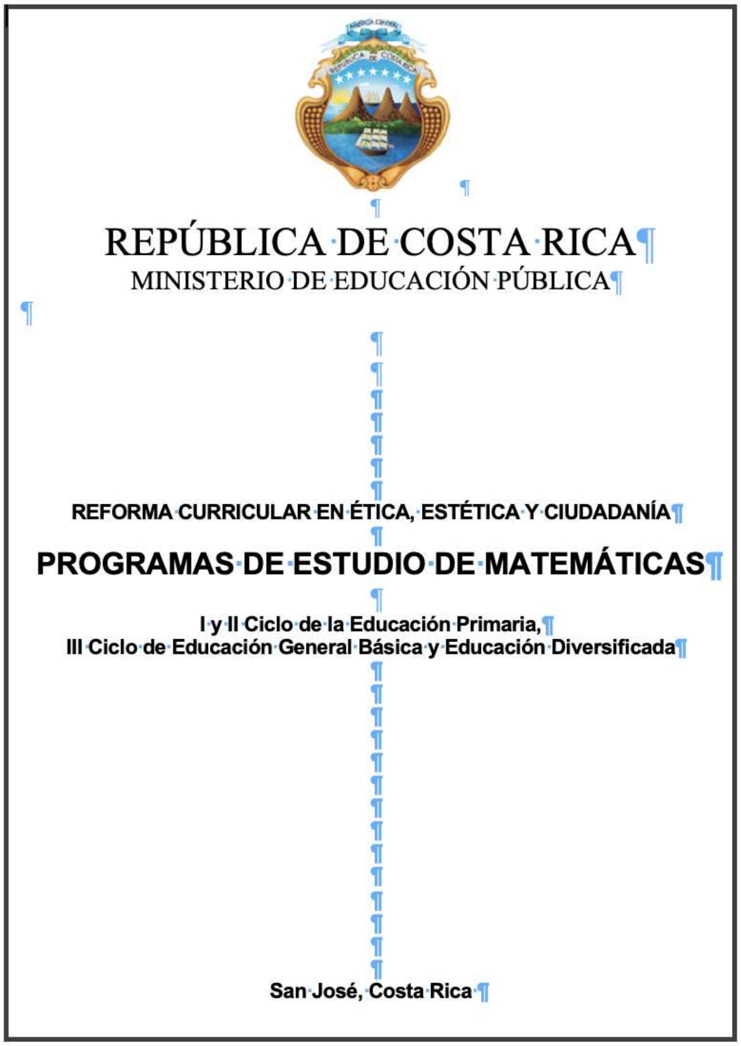
COSTA RICA SCHOOL MATHEMATICS CURRICULUM 2012

Theoretical Foundations


REPÚBLICA DE COSTA RICA
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REFORMA CURRICULAR EN ÉTICA, ESTÉTICA Y CIUDADANÍA

PROGRAMAS DE ESTUDIO DE MATEMÁTICAS

I y II Ciclo de la Educación Primaria,
III Ciclo de Educación General Básica y Educación Diversificada

San José, Costa Rica

This document is a translation into English of the sections on theoretical and pedagogical foundations that are included in the official school mathematics curriculum for grades 1 to 12 of Costa Rica. The vast majority of these texts were written by [Angel Ruiz](#).

It does not include the endnotes that were oriented to researchers about certain theoretical details.

The complete curriculum in Spanish can be viewed or downloaded at https://www.mep.go.cr/sites/default/files/programadeestudio/programas/matematica.pdf

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GENERAL INTRODUCTION

Mathematics has always occupied a relevant place in people's knowledge and culture. Together with other wisdom and human practices, it has been privileged to define models of reality, structures of knowledge, and propose cultural meanings. The vertiginous progress of sciences and modern technologies has seen a more intense participation of Mathematics, and at the same time it has been a powerful factor in its advance. Mathematics is present in Information and Communication Sciences, in Physics, Chemistry, in space technologies, nanotechnology, weather forecasting and the calculation of risks and benefits of financial entities. Mathematics is a key part of both the intellectual devices to scrutinize the origin of the universe and of life, as well as the construction of sophisticated artifacts that we use daily. Mathematics, too, has been a fundamental element in the formation of a rational and critical spirit, a bastion of logical argumentation and the justification of reasoning. Mental rigor and intellectual insight have been abilities cultivated through Mathematics.

There has been an intimate relationship between mathematical constructions and their teaching, since in many ways the nature of Mathematics is exhibited, based, and included in those actions related to its teaching and learning. The teaching of Mathematics, however, has its own meaning and characteristics independent of Mathematics, which have to do with the specific purposes it has. The mathematics preparation that is provided in the school classrooms must find its general meaning in the development of the abilities of the individuals to intervene in a better way in life.

Problem solving as a main focus of the curriculum

This curriculum assumes as its main objective the search for the strengthening of higher-order cognitive capacities to address the challenges of a modern society, where information, knowledge and the demand for cognitive capacities are strongly invoked. Developing this purpose supposes at least two things: on the one hand, that each student assumes a commitment to the construction of their learning, and on the other, that there is a crucial teaching action to generate learning in the quantities and qualities that the current scenario implies. Learning to pose and solve problems and specially to use them in the organization of the lessons is adopted as the central strategy to generate these capacities. The intellectual challenge is substantial, and a nutrient for intelligent and motivating classroom work.

In this curriculum, emphasis will be placed on working with problems associated with real, physical, social, and cultural environments, or that can be imagined as such. It is assumed that using this type of problems is a powerful source for the construction of learning in Mathematics. When placed in real contexts, the formulation and resolution of problems leads directly to the identification, use and construction of mathematical models.

Problem solving as a pedagogical strategy will be underlined here as a substratum of a classroom action style. For the learning of knowledge within the lesson, an introduction of the new topics is proposed that considers four steps or central moments:

(1) proposal of a problem,
(2) independent student work,
(3) interactive and communicative discussion,
(4) closure.

This sequence can be done within a lesson or a collection of them according to the topic or the school year. This style is opposed to the one that works on mathematical topics in the abstract, offers examples and routine practice and at the end, as an appendix, exercises, or contextualized problems. It is not a prescription to be followed mechanically, since its design and implementation depend on the conditions in which learning is planned.

Using problems is proposed as a constant in all phases of classroom action, including that of reinforcement, mobilization and application of learned knowledge.

Although the use of problems in real contexts is encouraged, abstract ones are considered very important. And even more, what is ultimately intended is the construction of capacities for the use of mathematical objects whose nature is abstract. The assumed strategy intends to provide a pedagogical foundation for the transition from the concrete to the abstract.

Basic concepts: abilities, competence and processes

The organization of the study program is carried out through five mathematical areas: *Number, Geometry*, *Measurement*, *Relations and Algebra,* and *Statistics and Probability*.

Mathematical knowledge is the basis of these programs. However, an approach based not only on mathematical content is adopted. What is intended is the development of greater capacities of citizens to face the challenges of the world in which they are a part. The vertiginous development of knowledge and its accelerated rate of change lead to a reformulation of programs, materials, texts, material, and human resources that strongly transform the teaching action and the organization of the lesson. A logic of knowledge in context, of learning to learn, is developed. Capacities are assumed as central. In the first place, capacities associated to mathematical areas and referred to as *specific abilities* were selected. Second, the generalization of these specific skills, *general abilities,* to be developed in an educational cycle were selected. Finally, and only as a general perspective, *mathematical competence* is developed. To carry out these purposes within the curriculum, the quantity and quality of educational content must be modulated based on the progress with respect to these capacities.

Mathematical competence is interpreted here as an ability to use Mathematics to understand and act on various real contexts, and underlines a relationship of this discipline with physical and sociocultural environments. A privileged place is thus provided for problem posing and solving. In this view mathematical competence is defined by a powerful practical sense. Embracing the meaning of mathematical literacy in this way has important implications for this school curriculum.

Mathematical competence, however, does not organize study plans. Mathematical competence and higher order-cognitive capacities are developed from daily activities in the classroom for the achievement of specific and general abilities (associated with mathematical areas). Mathematical knowledge and the related learning expectations are the starting point in each cycle and school year. They constitute the immediate teacher contact with the study plan of each school year. This is fundamental because it demands that the program not distance itself from the current preparation of teachers in the country and the dominant tradition in terms of the curriculum. There is full familiarity with the mathematical areas.

The mastery of abilities in a mathematical area and the development of mathematical competence is proposed to be carried out through pedagogical mediation. Several strategies are developed concerning the organization of lessons and mathematical tasks, and direct teaching action in the classroom. Among them, ensuring that *mathematical processes* are carried out in the classroom action, that is, transversal activities that are associated with abilities present in each area to understand and use knowledge, supporting the development of mathematical competence.

Five central processes are proposed here: *To reason and argue, To pose and solve problems, To connect, To communicate* and *To represent.* They are forms of cognitive action that can generate capacities. The selection and conceptualization of these processes orders and defines the role to be given to mathematical capacities (for example, closely associating problem solving and modeling), and facilitates the implementation of high-order transversal cognitive actions in the classroom. It is accepted as a premise that its constant realization in all school years encourages the generation of progress in mathematical competence. Actions are indicated in the syllabus and "study plan" to be carried out in each educational cycle.

Quality Mathematics with depth

There is a growing social demand that people be able to perform operations and mathematical processes of greater complexity. This refers to mathematical capacities associated with problem solving, the application of concepts and procedures, mathematization or modeling, as well as higher levels of mathematical justification and argumentation. The management of simple mathematical concepts, processes or activities, simple or routine procedures and of lesser mental demand, must be a subordinate part of higher-order mathematical actions. Both in the learning stage, and in the mobilization and application of learning, it is advocated to work with problems that have different levels of complexity.

The in-depth mastery of some topics generates capacities to be able to learn other topics more easily (even without the teaching context. On the contrary, superficial contact with many topics does not allow significant learning and rather becomes an obstacle to learning progress.

In addition to working with problems with different levels of complexity, it is necessary to introduce mathematical content that plays a crucial role in modern school education. For example, there are topics of coordinate geometry and plane transformations that, in addition to including a modern vision of Geometry, favor the treatment of other mathematical concepts and procedures, providing instruments to be able to use Mathematics in various contexts. An adequate treatment of relations and functions is another purpose that must be emphasized and properly cultivated from the beginning of school education, since these are central to a modern mathematical education. Statistics and Probability are a mandatory part of the knowledge that a citizen must have in our scenario.

Privileging the depth in the treatment of school content over its breadth requires providing adequate time for learning. And that has consequences on the quantity of contents in the study plans for each grade. It is often wrong to think that when a study plan exhibits a wide collection of contents, the best educational instrument is offered to the country. The error is made in two senses. Mastery of them is neither provoked nor are mathematical capacities affirmed, which are not only essential for professional lives associated with science but for the construction of a critical and rational citizenship. In this way, it becomes a bad instrument, far from the classroom reality and from the effective possibilities of positively influencing learning.

Promoting breadth by itself regardless of depth is an error, but another almost symmetrical error would be the reduction of content to an undesirable minimum that can only harm the education that the country must provide to its citizens.

The purpose of offering quality Mathematics is to provide citizens with the best tools to enhance their living conditions in this historical context. In this sense, it seeks to provide all social and cultural sectors with a modern and solid Mathematics program that promotes *social equity.* This curriculum should be able to be implemented throughout the country. The State must assume the actions required to ensure this equity everywhere.

Vertical integration of study plans

The knowledge and expectations of learning about them are organized in the curriculum in an integrated way from the first to the last year.

There is epistemological and pedagogical support for this decision. Mathematics is not a dispersed and disjointed collection of specific concepts and procedures. They are integrated from general ideas and methods whose construction and expansion have been the result of mathematical tasks. In each mathematical area fundamental general ideas can be pointed out and the different associated dimensions or reconstructions that they have been given over different historical moments can be described. Similarly, new ideas and methods are built every day through mathematical communities. However, not all general mathematical ideas and methods are relevant for introduction to school curricula. What must be introduced are the basic ideas that underpin the mathematical edifice and whose mastery generates the capacities to access others, and those that, when introduced, can foster relevant conditions for the citizen. These ideas and methods are accurately included in these programs. This basic and general character makes it important that they are present at the different levels of the school curriculum, which must be done in different modalities, depths, and approaches.

There are other elementary reasons: With this perspective, more flexibility can be provided, since it is appropriate that the specific curricular goals are not established based on borders rigidly marked by educational levels or cycles.

This approach at the same time allows enriching the meaning of many of the topics, purposes, and potentialities of the same, by visualizing them in all their dimensions from the beginning of school life. In this way the topics in each year can be seen as particular or preliminary cases of more general ideas. For example, introducing regularities and patterns to prepare functions, manipulation of symbols to prepare the handling of algebraic expressions, representations in coordinates earlier to show meanings of geometric figures, etc.

This perspective, therefore, offers the teacher a foundation to make strategic decisions about the crucial moments in which some elements are introduced that will favor their learning in later years. Put in another more practical way, it proposes the design of mathematical tasks with a different vision nurtured by the long term. In addition, this offers better opportunities for coordination and collaboration between teachers of different cycles and educational levels on the topics to be developed.

With this vision, it was decided to distribute the knowledge and abilities of the curriculum in mathematical areas for all years from the first to the last. In the same way, the mathematical processes and the disciplinary axes or transversal emphases that are adopted here intervene in the entire curriculum.

Finally, although a vertical integrative approach is promoted here, the differences in the different educational cycles are considered to define its goals, in particular the difference between the second and third. The detailed organization of the programs, for their operational implementation, is done through the cycles that the Costa Rican educational system has.

Historical sense and adaptation to the national educational context

The school curriculum is just a means to an end: better learning. It has a historical meaning, that is, it is *temporary* and must be conceived for a precise historical stage. When the conditions are different due to many factors (even non-educational), it must be transformed.

This is also associated with the approach assumed here. If it were based strictly on content, its logic would not be conditioned so much by the context and its protagonists. Decisions regarding the quantity and qualities of knowledge, the selection of capacities and methods and management must be made based on these elements.

One cannot fail to keep in mind that the curriculum must be implemented (taught and learned), and this refers to the main protagonists who put it into practice: teachers and students, as well as the institutions that participate. An estrangement from its realities can only contribute to emptiness and sterility.

Not everyone can take advantage of a curriculum in the same way, and, although the State must offer a quality curriculum for all under an equity criterion, it must have enough versatility to offer different options.

The existence of diverse intelligences and talents must be taken into account by a flexible national curriculum. Over time, the Costa Rican State must generate alternative instruments to compensate for the extra-educational weaknesses that may exist and at the same time encourage the education of talented students in different areas. Although the curriculum is not responsible for fully responding to these needs, it does try to integrate them in some way. The place that has been selected to do this is precisely at work with different levels of depth of the content to be learned. In this way, mathematical content can be modulated according to different talents.

It should be underlined that in the quantity, qualities and logic of the selected content and its presentation in the study plan for each school grade there are implicit curricular decisions, which take into account the conditions of the national context.

A good resume is necessary, but it is not enough.

Disciplinary axes

Here, five disciplinary axes are adopted that cut across the curriculum and strengthen the curriculum:

- Problem solving as the main methodological strategy.
- Active contextualization as a special pedagogical component.
- The intelligent and visionary use of digital technologies.
- The promotion of positive attitudes and beliefs about Mathematics.
- The use of the history of Mathematics.

The first two axes are assumed as articulators, by which is meant that they not only permeate all the programs but also serve to structure and articulate the other axes and the different activities involved in its implementation.

Problem solving corresponds to the need to assume standards whose suitability for Mathematics Education has been widely proven on an international scale. The proposed contextualization seeks to strengthen an active student role committed to their learning, emphasizing the identification, use and design of mathematical models suitable for each educational level. There is an association between these two axes that precisely obeys the focus of this curriculum: problem solving in real contexts. And it is consistent with the selection and conceptualization of the mathematical process *Posing and solving problems.*

The use of technology assumes the contemporary trends of intense expansion of digital instruments and the need to configure a lucid and adequate use. The use of the history of Mathematics responds to purposes to provide a human face to Mathematics and achieve a synergistic action of the other axes.

The explicit incorporation of the search for positive attitudes and beliefs about Mathematics is in tune with the premise that attitudinal and socio-affective spaces are crucial for learning. Here are five attitudes to develop:

- Perseverance.
- Confidence in the usefulness of Mathematics.
- Active and collaborative participation.
- Self-esteem in relation to the mastery of Mathematics.
- Respect, appreciate and enjoy Mathematics.

These axes participate in the curriculum in different ways and emphases according to the mathematical area and the educational levels. These disciplinary axes are operationalized in the curriculum in a precise way through content, various indications, and suggestions.

The mathematics curriculum and the goals of Costa Rican education

This curriculum is part of the more general goals of Costa Rican education:

a) The formation of citizens who love the country, and are aware of their duties, rights, and fundamental freedoms, with a deep sense of responsibility and respect for human dignity.
b) Contribute to the development of the human personality.
c) Train citizens for a democracy in which the interests of the individual are reconciled with those of the community.
d) Stimulate the development of solidarity and human understanding.
e) Preserve and expand the cultural heritage, imparting knowledge about the history of man, the great works of literature and the fundamental philosophical concepts.

It is intended to affirm a vocation of mathematical competence especially associated with the construction of essential citizen capacities for the progress of the nation. It is not about training minds to be able to carry out exclusively limited purposes such as the mastery of sophisticated techniques of mathematical proofs or the construction of tremendously abstract structures far from the environment, or for the ethereal and private enjoyment of knowledge. It is sought through Mathematics to support citizen understanding and intervention on various physical, social, professional, scientific, and cultural contexts, and therefore provide individuals with conditions to be able to contribute to the progress of the country, within a spirit of responsibility and respect.

The strengthening of attitudes, beliefs, and positive values about Mathematics as a transversal disciplinary axis is explicitly assumed, which not only contributes to the development of the individual personality of those who participate in the educational action, but also widens the space of values and attitudes in general, such as solidarity and cooperative action.

The use of the history of Mathematics offers valuable opportunities to strengthen the cultural heritage of the human species, to establish a connection with the humanities and to get in touch with more general philosophical perspectives. These perspectives seek to give Mathematics a necessary human face not only to capture student interest but also to strengthen the possibilities that this discipline as a whole contributes more to the collective progress of the country.

Problem solving as a pedagogical strategy converges essential principles of constructivism, a philosophical premise of the national educational policy, such as the autonomous student construction of learning, but in an even more vigorous and effective way, since an independent and committed action of the student is a fundamental subject in the classroom action. The use of problems as generators of the organization of the lessons offers valuable opportunities to connect with the needs of our country, to cultivate reason and to develop a humanistic vision. Associating Mathematics to real contexts, promoting positive attitudes and beliefs, broadening the place of reason and promoting a historical and social vision of Mathematics underpin a comprehensive education where the strictly technical dimensions of Mathematics converge with those socio-affective influenced by personal environments and cultural.

In accordance with the Costa Rican educational policy, transversal axes have been incorporated here into school subjects:

- Environmental Culture for Sustainable Development
- Comprehensive Sexuality Education
- Education for health
- Experience of Human Rights for Democracy and Peace

The way in which they have been introduced is through problems and various learning activities that have been selected for classroom action. However, not all mathematical areas can be introduced in the same way. Also, the pedagogical strategies that are permanently proposed in the collaborative construction of mathematical learning and rigor in thought favor the formation of a socially responsible and critical citizenry, which constitutes a nutrient of democratic life.

The purpose of incorporating these transversal axes is strongly favored by this curricular approach that underpins the relationship between the teaching and learning of Mathematics with the social and cultural environments, which are naturally incorporated into the study plans.

The structure of the curriculum

This curriculum has several parts:

I. Basics
II. Axes
III. Classroom management and pedagogical planning
IV. Methodology
V. Assessment
VI. Syllabus for each educational cycle
VII. Other elements.

In *Foundations* the main terms and concepts that support the curriculum are consigned and in *Axes* the curricular disciplinary axes are described. *Classroom pedagogical management and planning* includes general indications for all the cycles on these topics. *Methodology* includes numerous general indications for all educational cycles on styles for organizing lessons, on areas and processes, on attitudes and beliefs, on the use of technology and the history of Mathematics. In *Assessment*, indications and general principles are provided.

The syllabus is organized through the educational cycles of the Costa Rican educational system. In this part are the mathematical knowledge and abilities as well as numerous additional specific indications that immediately accompany specific knowledge and skills on method, management, and evaluation. In addition, suggestions are incorporated (always per cycle) on mathematical processes, uses of technologies and strengthening of positive attitudes towards Mathematics. These more specific indications are essential to delimit and exemplify these contents. Finally, some indications on assessment are introduced for each cycle in each area.

The last part, *Other elements,* includes a proposed thematic sequence for the implementation of the topics of the mathematical areas for each school year (where it is pertinent), a glossary (with key terms used), a knowledge table (which allows global visualization of the study plan in terms of content) and the general bibliography that was used.

With the purpose of creating a functional and practical document, that is easy to read, it was decided not to include throughout the text the large number of references of results or experiences that were used in this curricular design. However, the pertinent documents consulted are included in the final bibliography and also some notes are placed at the end with references for use by teachers or researchers.

This curriculum exhibits a deep integration of its different components (theoretical, pedagogical, and practical), coherence between foundations and curriculum, as well as an expressed vocation to support the teacher. The vision is assumed that these elements offer better possibilities for their implementation, thus nurturing the progress of Mathematics Education in the country.

I. FOUNDATIONS

The most general perspective sought in this curriculum is to provide the youth of Costa Rica with a mathematical preparation that allows them to address the challenges they face in the current scenario with intelligence, relevance, responsibility, and success, creating means to promote a more educated society, more inclusive and more democratic.

The focus that this curriculum assumes is the cultivation of problem solving in real contexts. It is a question of a pedagogical mediation that adopts fundamental constructivist premises, in accordance with the educational policy approved by the country, one that emphasizes the active construction by the students of their learning. This approach pursues the development of student abilities directly associated with mathematical areas to be generated in short periods, as well as others of a higher cognitive order of a transversal nature with a perspective of medium and long terms.

The description of the meaning of this approach is developed in several sections:

- The curriculum is organized by mathematics areas and skills.
- A perspective: Mathematical competence.
- Mathematical processes.
- Pedagogical mediation: A key to the development of higher-order cognitive capacities.
- Problem solving.

The curriculum is organized by mathematical areas and abilities

This curriculum breaks away from strictly content-based approaches. However, the role of content is not underestimated. On the contrary, the basis that organizes the study plans in each cycle and school grade is mathematical knowledge and the abilities around them that are expected to be learned. Five mathematical areas are used here:

- *Number*
- *Measurement*
- *Geometry*
- *Relations and Algebra*
- *Statistic and Probability*

The first introduces number, number systems, operations, and calculations. Their properties are studied but within an eminently pragmatic perspective that emphasizes student action: The calculation and use of numbers in the representation and manipulation of the world.

Geometry refers to the study of the characteristics of geometric figures and the relationships between them, geometric modeling, and spatial visualization, which allow enhancing the processes of visualization, classification, construction, and argumentation. The movement of geometric shapes is emphasized.

Measurements raises the understanding and manipulation of units, systems and processes of measurement of space and time, and the use of tools and formulas to carry out the measurements. This area plays a very important role, which has traditionally been confined to Primary education. In that way in Secondary school a greater domain in calculations, approximations, and estimations in measurement was ignored, as well as the contextualized treatment of mathematical topics, for example in Geometry, Statistics and Functions.

Statistics and Probability. This area includes two major topics. On the one hand, the identification, organization, and presentation of information, which is associated with descriptive statistics, and on the other, Probability, which refers to the study of uncertainty and chance.

Relations and Algebra refers to various topics such as the study of patterns and relations of different types (numerical, geometric), functions (seen as relations between variables), as well as the handling of expressions and symbolic relations, equations and inequalities, as a means of promote processes of generalization and symbolization. Algebra is not seen only as manipulation of symbolic expressions or procedures to solve equations but as a powerful means to represent numerical and geometric situations. Equations and inequalities, for example, can be better appreciated as representations of relations of variables whose paths (or domains of application) can be many. Sometimes they can be integers, rational or real numbers, geometric shapes or properties of space. In this way, algebraic expressions can represent regularities and patterns in many circumstances.

The participation of the same areas from the beginning of Primary to the end of Secondary especially strengthens the vertical integration of the curriculum.

Here the term "specific ability" is used as an ability or *know-how* in relation to a mathematical object (concept or procedure), for instance:

- To recognize quantities less than 100.
- To know the names of numbers less than 100.
- To perform sums of natural numbers, without grouping, with totals less than 100.

In this approach, abilities are always associated with a mathematical area (arithmetic abilities, geometric abilities, algebraic abilities, etc.), and thus, are associated with mathematical knowledge. “Specific” abilities are proposed to be developed in relatively short times. They should not be seen as abilities that one has or not (or ends achieved or not) but as *learning expectations* that can be achieved gradually. The specific abilities could be visualized as specific curricular objectives, although not in the way proposed by behaviorism (“operational objectives”, observable, measurable, quantifiable). These abilities around knowledge can be grouped, so that they are worked on in this way both in the classroom action and in the assessment. This characteristic, which is explicitly sought to be promoted in this curriculum, distances the sense of “ability” from what is usually considered as the “operational objective”.

Specific abilities can be seen as special cases of “general abilities” (for example, “doing addition with natural numbers” is a generalization of “add with natural numbers up to 100”).

In the curriculum, the specific abilities have been placed as learning expectations for each school grade, while the general abilities as learning expectations for each educational cycle. In both cases they are associated with mathematical areas.

A perspective: Mathematical competence

For a long time, educational curricula were proposed that were essentially lists of topics without much relation to learning or to the conditions of classroom action. For several decades there has been a notable curricular development in the world that has been progressively abandoning this content-based approach. One way that seeks to break with these schemes is the perspective of competence. The basic idea has been to place as the most general purpose the generation of capacities in different terms, strengthening its connection with life. In this perspective, content learning is seen as a function of these capacities.

The concept of competence has been used in various ways in the international community. In this curriculum, the meaning used by the *International Learning Assessment Program* of the Organization for Economic Co-operation and Development (PISA and OECD, respectively) is accepted. The theoretical framework that supports this program constitutes a central reference on the world stage for educational purposes. In this curriculum, its use is assumed as appropriate to what is pertinent.

Competence means:

> (...) the ability of students to apply knowledge and capacities, and to analyze, reason and communicate effectively when posing, solving and interpreting problems related to different situations.

> It is measured in a continuous way, not as something that a person has or does not have. (...) the variable character is a fundamental trait. An educated person has various capacities, and there is no clear line between someone who is fully competent and someone who is not. (OECD, 2005, p. 23).

In the same way, the following definition of mathematical competence is selected:

> (...) an ability of the individual to formulate, use and interpret Mathematics in a variety of contexts. It includes reasoning mathematically and using concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It helps individuals to recognize the role of Mathematics in the world and to make well-founded judgments and decisions necessary for constructive, engaged, and reflective citizens. (OECD, 2010, p. 4).

This definition establishes that *mathematical competence is* formulated in relation to the use of Mathematics to describe, understand and act in various contexts of their reality (personal, physical, social, cultural).

This way of understanding competence makes it possible to contribute to the more general goals of Costa Rican education in the "development of the human personality", in the participation of citizens with a "sense of responsibility", understanding and "respect", which allows reconciling " interests with the community" (a foundation of democratic life) and cultivating a reflection that supports the rational understanding of the diverse cultural and social contexts, of the ideas and achievements that constitute human history.

Mathematics preparation affirms here the purpose of generating mathematical competence with these characteristics, thus impacting school programs. Not only does it underpin a privileged relationship between Mathematics and real environments (it seeks to have an impact on the world), but it is also associated with problem solving in the general sense of that expression, thus favoring student capacities to propose and design strategies to solve problems, which is possible to promote also in several other educational subjects. In different ways each individual faces problems in various existential contexts, and all school subjects could have problem solving as a strategy, although of course with different instruments. In the case of Mathematics, the scope of the abstract and the pre-eminence or density of the problems force an educational and curricular strategy of another magnitude.

It is here a specific vision of the meaning of mathematical competence, since the perspective could be different. For example, it could be assumed that the mathematical competence that is sought to provoke in school education is the mastery of mathematical structures or formalisms. It could also be assumed that a person is more mathematically competent than another if they know a greater quantity of mathematical contents, which within the study plans would give another particular perspective (for example, always seek to introduce the greatest possible quantity of contents). This is not accepted here. Mathematical competence has in this perspective a very important practical sense. The direction chosen here is based on the general purpose of mathematical preparation that seeks to provide citizens with means that contribute to their participation in their context in a positive, intelligent, reflective, critical and responsible manner.

This mathematical competence will make sense of many global decisions that are found explicitly or implicitly in the curriculum. In the first place, it provides meaning and coherence to the various parts of it, is a powerful instrument to establish its general goals and its borders, nourishes it and gives it direction. It is a means to establish strategic curricular disciplinary axes, offers criteria for the presence or absence of content and motivates an approach to classroom action that privileges problem solving as a pedagogical strategy, especially in real contexts, strengthens the participation of the identification, construction and use of models, gives meaning to the strengthening of areas such as Statistics and Probability, and nurtures the role of technologies.

Mathematical processes

Mathematical *processes* are understood here as cognitive activities (or types of activities) that people carry out in different mathematical areas and that are associated with capacities for understanding and using knowledge. The systematic realization of these transversal processes in the classroom action supports the progress of various dimensions of mathematical competence.

It is worth saying that these mathematical processes are not capacities but support their development, and also have numerous intersections with each other (one-to-one).

Five processes

The following processes have been selected as central:

- *To reason and argue*
- *To pose and solve problems*
- *To communicate*
- *To connect*
- *To represent*

The description of these five processes is given below.

To reason and argue

These are mental activities that appear transversally in all areas of the curriculum and that trigger typical forms of mathematical thinking: deduction, induction, analytical comparison, generalization, justifications, proofs, use of examples and counterexamples. It seeks to develop capacities to allow the understanding of what a justification or proof in Mathematics is, to develop and discuss mathematical arguments, to formulate and analyze mathematical conjectures, and to use mathematical formulas or methods that allow the understanding or development of present information.

To pose and solve problems

This refers to the formulation of problems and the design of strategies to solve them. Here a privileged place will be given to problems in real contexts.

It seeks to enhance capacities to identify, formulate and solve problems in various personal, community or scientific contexts, inside and outside of Mathematics. Then, these are capacities to determine the most appropriate strategies and methods when facing a problem, to assess the relevance and adequacy of the available methods and the mathematical results originally obtained, as well as the ability to evaluate and control the development of their work in problem solving.

The desired emphasis on real contexts also promotes an association with the development of cognitive capacities to identify, formulate, design, develop and contrast mathematical models of the environment with diverse complexity.

To communicate

To communicate is the oral expression or the communication of visual or written of mathematical ideas, results and arguments to the teacher or to other students.

This process seeks to enhance the ability to express mathematical ideas and their applications using mathematical language (syntax and semantic rules) in writing and orally to other students, teachers and the educational community. It aims to develop capacities to record and express with mathematical precision the ideas, arguments and procedures used, as well as the conclusions reached, as well as to identify, interpret and analyze written or oral mathematical expressions made by other people.

Due to the great presence of symbolizations in Mathematics, it is sometimes thought that oral and written communication are not relevant. It is common to not include them in the classroom action or in the forms of assessment. However, oral and written communication are central processes for the generation of mathematical competence, since they allow mathematical ideas to be clarified, shared, revealing different dimensions, and expanding active student participation.

To connect

This transversal process aims to prepare students in the first place in obtaining relationships between the different mathematical areas, and it derives from the central characteristics of mathematical tasks: their integrated nature. Professional mathematicians apply mathematical methods and objects from one area to another. Although Mathematics has evolved into different disciplines or areas, they have become integrated over time. This integration is of such a level and the flow of relationships from one side to another is so great that not insisting on those connections and that unified character would lose the proper understanding of what Mathematics is.

With this multiplicity of connections, the limits and meaning of many of the mathematical objects are better understood. In the school context, training and developing the ability to make connections can be done at all educational levels without great difficulty.

This process seeks to cultivate the relationships between the different parts of school Mathematics, in addition to the development of actions to identify within non-mathematical situations those in which a mathematical treatment is possible. And in the same way, it seeks to motivate connections with other subject matter and with different contexts.

To represent

Here the aim is to promote the recognition, interpretation, and manipulation of multiple representations that mathematical notions have (graphic, numerical, visual, symbolic, tabular).

The process seeks to promote the ability to develop and use mathematical representations that serve in the registration and organization of mathematical objects, to interpret and model mathematical situations, and to manipulate different representations of mathematical objects. It also proposes developing capacities to be able to translate a representation in terms of others, understanding the advantages or disadvantages (or the scope) of each representation in a given situation.

How do the processes work?

Due to the meaning given here to mathematical competence, it will be sought that most of the activities develop the process of *To pose and solve problems.* The place of the *To reason and argue* process is also very broad, since it is linked to other central characteristics of mathematical thought. The other processes *To connect*, *To communicate* and *To represent*, always important, join the powerful participation of the first two. This makes the place of the first two processes more apparent. The five processes propose an explicit teaching action in their professional work in the classroom.

Mathematical processes have different intersections between them and that is why they act in a linked manner. In specific classroom circumstances, an activity that emphasizes *To pose and solve problems* can appeal to *To reason and argue*, *To represent*, *To connect* and *To communicate* in different ways. It is difficult to consider Mathematics separately from reasoning and mathematical argumentation. The precise way in which one process is associated with another is not the same in every mathematical circumstance. Sometimes, *To pose and solve problems* will be activated more linked to *To connect*, in others to *To communicate*, everything depends on the mathematical task.

Each central process indicated here could be seen as a synthesis of processes from other levels of specificity or domains of action, since in the subject (learner) all the processes (whether general or less general) are not activated in isolation. For example, when carrying out the process *To represent*, other more general (or basic) processes can participate, such as identifying, listing, ordering, classifying, etc.

Pedagogical mediation to develop higher-order cognitive capacities

In this curriculum, on the one hand, abilities associated with mathematical areas are proposed, and on the other, processes are proposed that support the generation of higher-order transversal cognitive capacities that will become evident little by little and especially in the medium and long terms. These dimensions are closely associated. The mathematical processes adopted are introduced from learning tasks in which the development of specific abilities is pursued.

The mediations between the development of specific abilities, higher-order cognitive capacities and mathematical competence are complex, difficult to identify and even more difficult to measure. The mastering of a specific ability can occur in different degrees or levels.

Mathematical knowledge or specific abilities do not by themselves generate broader higher-order cognitive capacities that nurture mathematical competence. It can be achieved by the way in which the mastery of these abilities is generated, that is, the way in which the classroom action is carried out, the pedagogical mediation. It is fundamental how the lesson or sequences of lessons are organized, the direct action of the teacher in the classroom activities and the quality of the cognitive demands that are provoked. Here the realization of the processes is central. If the lessons are always masterfully organized and without the active participation of each student, or if learning tasks that challenge their intelligence are not proposed, interest and active commitment are not provoked, the possibilities to motivate higher level mental actions are weakened. It is about, then, designing lessons with learning tasks that allow the development of processes. If at the beginning of the lesson it is proposed to identify, formulate, and solve a problem, the development (activation) of the mathematical process of *To pose and solve problems* will be supported. From this, you can introduce the others.

It must be understood, however, that developing or activating a process is sometimes a complex task and that it cannot be applied at all times. The place and time should be selected carefully. On the other hand, it must be taken into account that in a single learning task it is possible to perform several processes at the same time or formulated in another way. The participation of a process intersects with others.

Not only the design of mathematical tasks is important. So is the direct teaching action during the proposed activities. Here intervention in each phase of the lesson will support the development (activation) of processes. The result will not be adequate if teachers exceed their participation (for example, rapidly solve problems on the blackboard as an initial doubt emerges, or give inappropriate instructions), are unduly absent from the activity, or do not correctly provide the necessary knowledge that would complete the student work. Direct teaching action is crucial.

In the middle of a learning activity aimed at achieving mastery of a specific ability, it is possible to dynamically introduce the mathematical processes that are relevant. For example, with suitable questions that provoke more implications or derivations to promote reasoning and argumentation, through connections with other mathematical areas, with the generation of the expression and communication of ideas in several planes, or with a motivation so that the mathematical entities that come into play can be represented in different ways. It is a matter to determine in a specific and practical way, as not all mathematical processes have the same weight in the development of a task, and sometimes some do not even appear.

The design of the mathematical task and the teaching leadership in the classroom are key instruments for these mathematical processes to be carried out. That involves careful lesson planning and design. Some tasks are better than others for this overlap of capacities and processes. Not only didactic research can provide examples and pedagogical results on how to carry out these actions in the classroom, but also and above all the research carried out by teachers in the different educational entities in a systematic and continuous manner allows to provide means to follow this strategy. In a general way, with a suitable perspective, certain experience and preparation, mathematical processes can be activated in almost any mathematical task oriented to the generation of a specific ability or a set of them.

Pedagogical mediation is the key so that in the activities the mastery of specific abilities is achieved and, in this way, higher-order capacities and mathematical competence are developed. It is necessary to keep always in mind the promotion of mathematical processes.

An example of how this could work in the classroom might be through a problem you can search for the generation of a specific ability like the following:

“Identify patterns or regularities in sequences and in tables of numbers less than 100”.

Time is offered to come up with solutions or strategies, and then the class is asked to communicate. This can push different kinds of solutions and then these solutions can be contrasted. By doing this, the mathematical process *To communicate* is carried out and in this contrast other activities associated with processes (*To reason and argue*) are brought into play. They may be asked to link this situation to some other area of Mathematics, and they may also be asked to justify various steps or be asked what would happen if some appropriate conditions of the initial situation were modified. In doing this, other mathematical processes can be involved.

By carrying out this type of process in the different mathematical areas, the student's ability to communicate and argue mathematically in an adequate way is increased.

Problem solving

Problem solving is as part of this pedagogical mediation and is where an essential meaning for the teaching and learning of Mathematics is found. It is a powerful instrument to achieve mastery of abilities, the activation of processes as well as the progress of mathematical competence.

Problem Solving: Purposes in the Curriculum

Problem solving is substantially associated with the nature of Mathematics, whether environmental or abstract problems. Intuiting, describing, posing, solving, and generalizing, mathematical problems define the activity of these professionals in socio-historical contexts where there are criteria and methods of communication and validation. There must be an explicit relationship between this nature and the teaching and learning actions. Not establishing these connections in the classroom action would mean the misunderstanding of a central meaning of Mathematics. However, moving from problem solving activities with more general mathematical tasks to classroom action cannot be done mechanically. There must be adaptation to the school environment.

Already placed in an educational context, problem solving must integrate at least two purposes:

- learning the methods or strategies to pose and solve problems,
- learning of mathematical content (concepts and procedures) through problem solving.

In the first purpose, the means (strategies, heuristics, methods) required by a problem (a mathematical action) are emphasized. Learning problem-solving techniques does not guarantee that a person can solve new and different problems, however, training in them favors the development of that ability. However, it would not be appropriate to conceive of the role of problem solving as reduced to training and obtaining skills in those techniques and methods, however rich they may be.

In the second purpose, what is proposed is a classroom action that allows generating mathematical learning in a specific context. This appeals to the design of tasks that serve to build learning within a lesson (or a sequence of them), thus promoting the performance of mathematical processes.

An essential premise is adopted here. Real problems, in which physical and sociocultural environments appear, play a crucial role. Using problems extracted from reality or that can be imagined as real promotes cognitive actions required for learning Mathematics. This happens for several reasons. On the one hand, because it is possible to arouse greater interest, provoke positive attitudes about Mathematics, involve people more in the construction of their learning and then stimulate various cognitive activities and the cultivation of mathematical competence. But it is not only a matter of motivation, in action to find, use or apply Mathematics within real contexts (well selected) contact with mathematical objects is promoted in their privileged relationship with the reality from which they emerged. Working in these diverse contexts favors a mathematization (using Mathematics to represent or model situations in the environment) that -although it must be adapted to the school environment- corresponds to those similar activities carried out in more general mathematical tasks.

In the approach that benefits here, the choice of a problem for the development of a lesson must be established by the purposes of learning mathematical knowledge and the educational development that is carried out, and not, for example, by the strategies or supposed techniques that needed for the solution. Although, without a doubt, there is a relationship between a problem rich in solution possibilities and the goals of good learning.

One of the aspects that we wish to underline in this vision is the importance of discovering, posing, and designing problems (and not just solving them), since in their lives people will be more exposed to circumstances in which problems are not formulated or possible Mathematics that can intervene are not visible or obvious.

Although emphasis will be placed here on the organization of the lesson, the purpose of developing competence in the resources and methods for solving problems is also incorporated into the different mathematical areas that organize these programs.

Problems

The contexts where a problem can emerge can be diverse: a health situation in the country, economic, environmental, cultural issues, as well as school, family, community, professional, scientific contexts. But a problem can also be designed from passages in the history of Mathematics, from an artistic representation where it is possible to find Mathematics, even a game, a puzzle, a video, etc.

A problem is an approach or a task that seeks to generate student questioning and action using mathematical concepts or methods, implying at least three things:

- to think about mathematical ideas without them having to have been explained in detail beforehand,
- to face problems without similar solutions being shown,
- that the mathematical concepts or procedures to be taught are closely associated with that context.

A problem must possess sufficient complexity to provoke non-simple cognitive action. If they are essentially routine actions, they will not be considered as problems. It can be put in the following terms: A mathematical task constitutes a problem if to solve it the subject (the learner) must use information in a new way. If the individual can immediately identify the necessary actions, it is a routine task. If a proposed mathematical task does not have these characteristics, it will be listed here as an exercise. A task can be an exercise or a problem depending on various educational circumstances. Addition with four-digit numbers can be a problem in Grade 1 and Grade 2, and an exercise in Grade 3.

The choice of problems posed in a real environment allows to enhance the application of mathematical concepts and methods, thus coupling with mathematical competence that has been defined as the ability to describe, understand, and act in different contexts (or situations) using Mathematics. Problem solving in real environments supports a perception of the usefulness of Mathematics.

There are also problems that by their nature do not admit a solution in a short time and others that may not have a solution. This type of problem offers opportunities to show some characteristics of mathematical construction. Mathematics does not have absolute truths. There are construction processes that can last a long time, etc.

It is appropriate to underline the importance of open-ended problems, that is, those that admit several solutions and approaches, and that can offer very valuable opportunities to introduce concepts and procedures, to organize the lesson or for homework through projects. When placed in real contexts, there are many opportunities to work with problems of this type.

Favoring problems in real contexts does not imply neglecting abstract problems. It is a general and flexible orientation that must be adapted with lucidity. There are mathematical areas and topics where work in real contexts does not make sense. But also, it should never be forgotten that Mathematics refers to general and abstract dimensions of reality (material and social). In essence, it is a practice that cultivates the abstract. In the educational action, what it is about is to build abstract objects based on a strategy that allows associations with the contexts at certain times to favor learning.

Abstract problems are crucial to bring different capacities and processes into play. In the abstract ones, for example, justification and demonstration, the use of mathematical language, rigorous abstract reasoning is developed. In the tasks confronted by professional mathematical communities, most of the problems that are addressed are abstract.

In the classroom, the moment, and the form to introduce a problem must constitute part of the educational planning. A problem does not always have to occupy a central role in the organization of the lesson, sometimes it can be introduced only in certain phases of the classroom action. On other occasions a problem can serve to reinforce learning. What is proposed here is the use of problems as a general orientation to be applied in a flexible way.

Regarding the general techniques or methods for the design of strategies to solve problems, four steps will be consigned as a guide, which are summarized in the following table.

Table 1. Problem solving steps

Steps or phases	**Action**
Step 1. Understanding the problem	Be clear about what the problem is about before you start solving it.
Step 2. Design	Consider various ways to solve the problem and select a specific method.
Step 3. Control	Monitor the process and decide when to abandon a path that is not successful.
Step 4. Review and verification	Review the resolution process and evaluate the response obtained.
Source: The curriculum authors	

There are several dimensions that participate in this procedure and that are relevant: resources, heuristics, beliefs, and metacognition or control. The first refer to the learned elements (correct or not) with which the person starts. Heuristics are the formal or informal strategies that occur or can occur to solve a problem, and there are beliefs about different dimensions involved. The evolution or monitoring of progress during problem solving and being aware of one's own abilities and limitations are essential at this stage and are identified with the so-called metacognitive strategies. In this sense, metacognition refers to the knowledge of our own cognitive process and the active control of the decisions and methods used in solving a problem. These general steps must be adjusted when working with problems in real contexts, as they require models.

Modeling

The identification, use and construction of mathematical models is a substantial part of the approach that is proposed to work with problems in real contexts. Models emerge whenever you must go to reality. A model is essentially a set of connected mathematical elements that represent a specific reality (explain, describe, allow predictions to be made). There may be several models of a reality with different degrees of representation of it. Identifying, building, or using a model of a real situation is a way of mathematizing that reality.

In this approach, a model is conceptualized in a broad and flexible way. It can be a diagram with arrows, a manipulative, a table, or a graph. A model refers to a specific situation, but when improved with cognitive actions it should be able to be used in other contexts.

What is proposed here is not only a training of students in the strategies for planning and building models themselves, but essentially using mathematical models and the actions involved in their construction and use to generate or reinforce learning. Specific knowledge and abilities can be built or applied through the actions that modeling offers. This school mathematization does not seek to give a final model and end the action there. It deals with the creation and successive use of models that are refined, adapted, and broaden their range of action. It refers to learning, to a constructive student action in which there is also teacher intervention.

The task of reworking and abstracting a model calls for more general and abstract mathematical activities. One could speak of two ways of mathematization. The first establishes and uses a model (it goes from the physical or social world to the symbolic mathematical world). The second theoretically refines and generalizes the results of the first (works in the symbolic and abstract world of Mathematics). Modeling will always appear as an integrated part of the process *To pose and solve Problems*.

The degree of complexity of the models generated will depend on the circumstances to which it refers, as well as the mathematical concepts and procedures involved, which must be adjusted at each educational grade.

The spirit of modeling resides in the identification, manipulation, design, and construction of mathematical models of authentic situations in the environment. This sense of reality is essential for learning. In very general terms, this action can be summarized in a few steps, which are listed in the following table.

Table 2. Steps in modeling

Steps	**Description**
Step 1. The Problem.	A problem that describes a situation of reality that must be modeled.
Step 2. Systematization.	A selection of the relevant objects, information and relationships of the problem that allow you to obtain a possible mathematical representation or idealization.
Step 3. Mathematical Model.	A translation of the objects and relationships from the previous step into mathematical language, in such a way that you obtain a model that represents what happens in reality.
Step 4. Solution.	Use of previous mathematical knowledge to be able to find the solution or solutions of the model proposed in the previous step. In this way it will be possible to obtain an approximation of the solution of the phenomenon that is being idealized in step 1.
Step 5. Interpretation.	Analysis of the results and conclusions considering the previous knowledge of the problem.
Step 6. Evaluation.	Verification in light of the mathematical results of the validity of the model and the predictive power that said model has on the original problem. For this process, comparison with observed data and/or theoretical knowledge or personal experience of the problem can be used.
Source: self made.	

The use of technologies

To work with problems takes on a vigorous perspective when it is done in real contexts and modeling is used. The use of digital technologies plays in the same direction, since it not only offers means that intervene as support (calculators or computers to simplify calculations, assess approximations, virtual environments), which allow visualizing dimensions that would otherwise be very difficult to incorporate into educational action (such as movement in Euclidean Geometry), but also to modify the meaning of some phases and objectives of problem solving. With technology it is possible to simulate real situations and reorganize the cognitive demands posed by a problem; redefine the strategies that can be designed.

The sense of contextualization and manipulation with real environments can be altered with technological means. In solving problems where technology can intervene, it is necessary to include other abilities and processes that are associated with the interactive relationship between knowledge, pedagogy and technology, conditions that are increasingly part of the generations of students who attend school (the manipulation of artifacts, special relationship with visual processes, multitasking, “social connectivity”, etc.). And this does not only refer to artifacts, the possibilities offered by the Internet for communication (where “distance” is relativized) allow working with problems (and with projects) in a completely different way than it would be done without these means.

For these types of considerations, the use of technologies should be assumed as a very important component for a curricular approach based on problem solving.

Different levels of problem complexity

It is essential to promote student confrontation with different levels of complexity in mathematical problems, since there is a directly proportional relationship between levels of complexity and the opportunities to carry out mathematical processes and nurture the progress of mathematical competence. The philosophy to be followed in the classroom varies in favor of emphasizing higher-order cognitive actions. A classroom action aimed at progressive confrontation with greater complexities is not consistent with educational styles that emphasize simple, repetitive actions or actions with little mental demand. Thus, the organization of the lesson must be rethought considering this vision.

This orientation, of course, does not arise outside of pedagogical considerations. It is not about looking for the complex for the complex itself, since not every complex issue is relevant. What it is about is that with relevant topics (mathematically and educationally) increasingly complex mathematical problems are confronted in a staggered manner. Through a greater depth in the topics, greater skills are forged to learn other contents.

Three levels of complexity are proposed here:

- Reproduction. In essence, this refers to relatively familiar exercises that demand the reproduction of knowledge already practiced. They appeal to knowledge of facts and representation of common problems, recognition of equivalent things, collection of mathematical objects or properties, routine procedures, application of standard algorithms, simple manipulation of expressions that have symbols, formulas, and simple computations.

  For example, at the end of basic general education:

  Solve the equation $8x - 2 = 15x + 9$.
  Find the average of the numbers 8, 13, 6, 15, 7.
  If 3,000 colones are deposited in a savings account and the bank offers 8 percent annual interest, calculate how much money in interest that deposited amount will earn after one year.
- Connection. This builds on capacities that are involved at the "Reproduction" level but goes further. It refers to solving problems that are not routine but are developed in environments familiar to the student. The interpretation has greater demands than in the representation group, and defines the situation. The connection is between the various elements, in particular, between different representations of the situation.

  For example, at the end of basic general education:

  > A bakery sells cakes in a circular shape and with the same thickness in 2 sizes: One with a diameter of 20 cm for 8000 colones, others with a diameter of 30 cm for 12,000 colones. Which cakes offer a better deal? Explain.

- Reflection. The significant element is reflection, carried out in environments that are newer and contain more elements than those that appear in the other level of complexity. The formulation and resolution of complex problems, the need for argumentation and justification, generalization, checking whether the results correspond to the initial conditions of the problem and the communication of these results are raised here. It requires the participation of several complex methods for its solution.

  An example, adapted from the 2003 PISA test:

  > A television documentary included a discussion of the predictability of earthquakes. Dr. Morales, a specialist in seismology, stated that in the next twenty years, the possibility of an earthquake occurring in the city of Santa Eulalia is two out of three. Which of the following best reflects the

meaning of the seismologist's statement?

A) 2/3 ×20 = 13.3 so between 13 and 14 years from now there will be an earthquake in the city of Santa Eulalia.

B) 2/3 is more than ½, so you can be sure that there will be an earthquake in the City of San Eulalia at some point in the next 20 years.

C) The probability that there will be an earthquake in the city of Santa Eulalia at some point in the next 20 years is greater than the probability that there will be no earthquake.

D) You cannot say what will happen, because no one can be sure when an earthquake will occur.

A curricular emphasis that assumes problem solving as its main focus cannot only deal with reproduction problems. The problems of connection or reflection are the ones that will put in motion more capacities. It is not about proposing most of the problems at these two levels, but rather that they are introduced according to the characteristics of the class, the moment in the sequence of lessons or the topic. This type of problem involves more processes. In teaching and learning, a strategy must be designed that uses problems at different levels of complexity, in a balanced way and attached to their contexts. That the complexity of a problem is related to the student must also be considered. For one group of students it may be connection, but for others it may be reflection. It is necessary to interpret the specific context in which it is found.

Problems, memorization and intellectual reflexes

Through problem solving, enriched and resized by real contexts and the use of technologies, the construction of learning is motivated. This is an approach that assumes that premise of the constructivist vision that sustains Costa Rican educational policy.

The learning of Mathematics is carried out in a progressive way from previous knowledge. Mastery and recall of some knowledge should be the basis for what follows. Only in this way can a coherent and structured body be formed in the mind. To face new problems, certain intellectual reflections are necessary that serve as assimilated and automated knowledge about procedures, algorithms or certain recurrent reasoning. This is essential to propose hypotheses, formulate strategies, identify the best procedures, and work routes, and imagine the ways to deal with a problem. And here precisely memorization and strengthening of reflexes in various dimensions is required. This should not be seen as a simplified way to build learning concepts or methods, but as an efficient way to access what has already been understood.

Opportunities should be promoted to perform synthesis, learn algorithms, and try to memorize common procedures or reasoning. For example, the basic algorithms of addition, subtraction, multiplication, division of various numbers, addition and multiplication tables, notable formulas, among others should be memorized.

It is necessary that the various topics be reviewed throughout the different years. But this should not be done through artificial general systematizations, but through new problems that intelligently recall the knowledge acquired in the past and thus raise other learning expectations.

II. AXES

Five disciplinary (curriculum) axes

Five transversal axes specific to Mathematics are assumed that enhance some relevant curricular dimensions for the effective teaching of this subject:

- Problem solving as the main methodological strategy.
- Active contextualization as a special pedagogical component.
- The intelligent and visionary use of digital technologies.
- The promotion of positive attitudes and beliefs about Mathematics.
- The use of the History of Mathematics.

These axes mean priorities here, so they must influence all the elements of the curriculum. These priorities are manifested in the selection of topics, in the general indications of management and method, in the indications and suggestions that accompany concepts and skills, in the planning proposal. It is sought that when implementing this curricular proposal, special importance is given to each of these axes, although not all of these axes generate an impact in the same way in each area or in each school year.

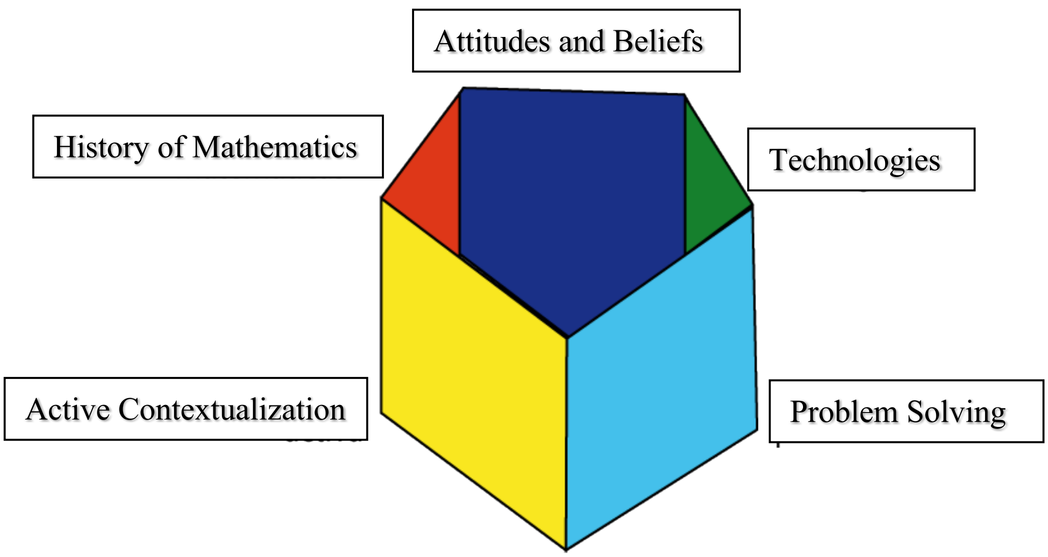

Figure 1. Disciplinary axes

The selection of these axes responds to conditions of the national educational context:

- In Costa Rica, problem solving has so far played a very small role and when it has been raised, it has been done in an abstract manner without being effectively carried out in the classroom and in most cases with an inadequate approach (an appendix to the discourse pedagogical or simple collections of strategies to solve problems).
- Although going to the environment for teaching has been considered, it has been done artificially, without provoking active student engagement.
- Similarly, technology has occupied a very weak place in official documents and in educational practice there have been serious distortions in its use.
- The theme of attitudes-beliefs and the use of history have been practically absent from educational programs and practice.

The axes seek to respond to existing weaknesses but also to position the Mathematics Education that is developed in the country with international standards. The action of the five axes in all educational years contributes to the vertical integration of the curriculum, especially through problem solving and active contextualization that seek to articulate the entire curriculum. The synergistic effect of these disciplinary axes seeks to favor quality mathematical preparation that helps generate competent, rational, responsible, and critical people for the construction of an educated, just and democratic society.

Problem solving and active contextualization

The main focus of this curriculum is problem solving in real world contexts. The most convenient way to promote its implementation is to place problem solving in the curriculum as disciplinary axes, do it in real contexts and also give these actions the greatest prominence.

The formulation that is selected to do this is through two emphases:

- Problem solving as the main methodological strategy.
- An active contextualization as a special pedagogical component.

This axis is consistent with the mathematical process *To pose and solve Problems.*

It is intended that these disciplinary axes dominate the teaching of concepts and abilities, and the design of mathematical tasks. The intense and appropriate use of technologies, the use of the History of Mathematics and the cultivation of positive attitudes about the subject must be done by favoring problems and an active contextualization. That is what using them as articulating axes refers to.

Here it is proposed that the two central purposes of problem solving take center stage. This means motivating the organization of classroom action through problems and promoting the learning of problem-solving strategies in the different mathematical areas. An active contextualization that stimulates student action is proposed, which requires the important use of models about the nearby reality.

The essential element of active contextualization is modelling. Mathematical objects can be contextualized in various ways. For example, a contextual introduction can be offered to address some mathematical concepts or procedures (“Note that in this classroom we have different shapes. Today we are going to study triangles”). Another way is to contextualize a mathematical situation (If Juan had 400 colones and spent 200 on candies, how many colones does he have left?). These contextualizations are useful in many educational circumstances, but they do not activate higher-order cognitive interests and actions or mathematical processes. They do not generate active student engagement. On the contrary, to arouse interest and participation, it is proposed to use problems in real contexts that provoke the construction or use of models. It is about designing problems taken from press information, from school, from the community, from the class, from the Internet. The same traditional “problems” that appear in many textbooks (normally as an appendix) can be enriched if they are placed in a modeling perspective and used to build higher cognitive capacities.

An active contextualization cannot be carried out in the same way in all areas, some lend themselves a lot, such as Statistics and Probability or Measurements. Likewise, real situations may allow you to use topics from various areas. It is not that all classroom problems are related to modelling, but that they are an important part of educational action. Solving problems in real contexts offers meanings, a sense of usefulness and diverse means to bring mathematical abilities and abilities into play, and allows scaffolding for the construction of learning from the concrete to the abstract.

Technologies

Digital technologies have had an extraordinary impact on both practice and research in Mathematics Education. Some of these resources are construction instruments and geometric experiences, data analysis, modeling and simulation, algebraic calculation, intelligent "tutors" and, increasingly, virtual spaces. Calculators, for example, offer the possibility of reducing routine calculations and concentrating efforts on the most significant reasoning or application processes for the domain of Mathematics. Computers allow the representation of mathematical concepts and procedures (mathematical objects that easily come to the world of the senses). These technologies not only favor multiple mathematical representation, but also extraordinary resources in the student-knowledge interaction, allowing an active involvement of the subject (learner) in their learning. The Internet, furthermore, is one of the most powerful vectors that directly or transversally amplifies the potential of the different technological instruments. Internet opens qualitatively different environments for teaching and learning, valuable resources for the construction of multiple pedagogical strategies. These technologies largely shape the contemporary reality of education.

Everything points to powerful changes in the future of educational topics and approaches. Many of the educational problems and tasks that were posed before these technologies have been qualitatively transformed, while new challenges and educational scenarios have appeared. Communication technologies have favored cooperative methods in the classroom and outside of it, building virtual spaces for communication and interaction, which can greatly transform the meaning of classroom work.

Technologies can be a powerful ally to enhance mathematical thinking. And it is precisely in solving problems in real environments where they can provide their benefits in the best way, in learning contexts that strengthen mathematical abilities and capacities. In this sense, they reinforce the implementation of the articulating disciplinary axes and add means to connect local Mathematics Education with dominant educational and cultural trends in the world. The current historical dynamic predicts a more intense penetration of all technologies in the social life of the country and the world. Study programs must prepare the population for this perspective.

The use of technologies, however, does not necessarily lead to the improvement of learning in Mathematics. Worse still, misuse can have a negative effect on learning. Technology must then be introduced in a relevant and precise way at the different educational grades and according to the material and human conditions existing in the national educational context.

Attitudes and beliefs

Motivation and interest and, in general, all affective dimensions are decisive in learning, which is why a comprehensive and humanistic vision of the teaching and learning of Mathematics is adopted here. Positive attitudes and beliefs towards the teaching and learning of Mathematics cannot be generated without the programs explicitly incorporating them and offering pedagogical means in that direction.

What are triggers of negative attitudes?

- Mathematics with a low level of demand for intelligent and creative action. Emphasis on mechanical repetition of simple procedures, meaningless memorization, or undemanding mental activities do not elicit empathy with Mathematics in most students.
- The separation or distance from student’s contexts.
- An organization of the lesson that does not favor the active and collaborative participation of students and teachers.
- Teaching separated from the cultural realities and technological means of modern society.
- Failure in exercises, problems and tests that generate a trail of low self-esteem and confidence.

Attitudes are closely linked to beliefs, which are even taken from the family and cultural spheres of society. Knowing these beliefs and reversing them towards others of greater positivity towards Mathematics should be a purpose to be incorporated at all educational levels. Identifying and transforming negative perceptions into positive ones should be part of the goals of an education anchored in the requirements of the society in which we live.

The attitudes to be promoted are:

Perseverance. One of the main attitudes that is sought to be promoted is the one that makes work, dedication, and persistence the means to approach Mathematics. Far from being a matter for gifted people, the truth is that mathematical skills are trained and developed.

Confidence in the usefulness of Mathematics. The claim to visualize the usefulness of these learnings for life is constant. With active contextualization, a valuable opportunity is offered to insightfully allow a link with student reality. The use of various digital technologies will also be an instrument to promote this understanding of the context and its closeness to the environment. In the same way, the progress in general mathematical competence to solve problems fosters this perception and attitude.

Active and collaborative participation. Getting each student to commit to building their own learning is a basic condition. The organization of the lesson should offer opportunities for active and interactive student participation.

Self-esteem in relation to the mastery of Mathematics. Many people feel that they are failing by not being able to successfully tackle mathematical tasks. With the presence of appropriate pedagogical supports and the existence of different levels of depth, it will be possible to shape personal demands to seek the development of this self-esteem.

To respect, appreciate and enjoy Mathematics. Although not all people will relate to Mathematics in the same way in their lives and not all will have the same abilities for handling it, it is important to develop a respect for its place in the knowledge and culture of humanity. For this, the elements that are contributed to the reflection are substantial, resorting to multiple media such as history, philosophy, engineering, the arts and other disciplines in which Mathematics is a fundamental part.

As with mathematical abilities, the progress of positive attitudes and beliefs towards Mathematics should be promoted in classroom action through teacher intervention. These purposes should be kept in mind in the various mathematical tasks. In the same way, contextualization, problem solving, technology and history expand the possibilities to appreciate the role of Mathematics.

History of Mathematics

Another of the disciplinary axes is the use of History of Mathematics in teaching.

Since mathematical objects correspond to the forms or instruments through which human beings organize the phenomena of their environment or respond to their challenges, it is important to know how this process has developed. Here the constructive, active, and dynamic aspects of mathematical objects are underlined. They are productions influenced by social contexts and in response to the conditions of their environment.

History is relevant to the perspective of this curriculum. Working with problems in real contexts seeks to reinvent or reconstruct the mathematical concepts and procedures that are studied in the classroom. Encountering the history of the construction of these mathematical objects favors their learning.

The History of Mathematics makes it possible to break with the scheme that Mathematics is a collection of axioms, theorems, proofs and where the essential thing is the logical clarity of its arguments. By placing mathematical objects in sociocultural contexts, it is possible to visualize the participation of heuristics, doubts, errors, misconceptions and even the existence of cognitive setbacks in some fields. Many mathematical ideas, such as the meaning of proof, rigor, evidence, etc, have always depended on contextual situations and the moment in history. The History of Mathematics underpins a humanist vision of Mathematics insofar as it underlines its character of sociocultural construction. Strengthening this approach contributes to training in accordance with the purposes of Costa Rican education.

Many more dimensions must be considered than those associated only with abstract mathematical results or developments. Individual or collective motivations, material and social conditions of a specific reality are also relevant. In the pedagogical action, Mathematics must be included in its context, and this appeals to History.

History will not be proposed here as a content to be evaluated, to offer flexibility when managing its introduction, in a national educational environment where the use of the History of Mathematics has not been a relevant part of school programs or pedagogical traditions.

The use of the History of Mathematics complements the other axes and allows them to be reinforced. There is a synergy. The most important impact of the use of this discipline, however, cannot be observed in relation to specific skills but rather in the medium and long terms since it is little by little that its limits and perspectives are understood.

The History of Mathematics not only offers very valuable resources for classroom action, but also enhances a perspective and an assessment of the discipline, which is relevant for effective learning and, even more so, for a cultural understanding of Mathematics, an imperative for every person in the scenario in which we live.

III. CLASSROOM MANAGEMENT AND PEDAGOGICAL PLANNING

The way in which the curriculum is interpreted and developed in the classroom is called curricular management, in which several factors intervene. On the one hand, the socioeconomic and cultural conditions, individual and collective, with strengths and weaknesses. On the other hand, the means, norms, and general resources provided by national educational institutions intervene, in addition to the educational environment itself (teachers of other subjects, administrative staff, support staff). It is in this context that the implementation is developed, whose central part is in the style with which the classroom action is organized.

This chapter will address educational management and planning in several sections:

- The organization of the lessons.
- General indications.

The organization of the lessons

As has been emphasized, pedagogical mediation is decisive for the achievement of the educational purposes that are proposed, which is why options are suggested here to pedagogically manage the action in Costa Rican classrooms.

A style for organizing lessons

Here is suggested a style for organizing the lessons where the multidirectionality of student and teacher contributions is supported, where there is an active participation and a collective construction of meanings, in order to activate mathematical processes that advance mathematical competence.

In the development of lessons there are two stages that can be distinguished for the purposes of teaching and learning:

- Stage 1: learning knowledge.
- Stage 2: the mobilization and application of knowledge.

The first stage is the one in which new knowledge is to be learned. The second occurs once the first has been completed and seeks to reinforce and expand the role of the learning achieved. This last stage can be carried out at any later time, not necessarily immediately after the first. In the first stage, it is appropriate to do it in a lesson or in a sequence of lessons.

A lesson organization style is proposed here where the introduction and learning of new knowledge is promoted following four central steps or moments:

1. Proposal or presentation of a problem.
2. Independent student work.
3. Interactive and communicative discussion.
4. Closing or closure.

1. Proposal of a problem.

In this first phase, a problem (contextualized when relevant), an initial challenge or an activity to provoke inquiry is placed as a starting point.

This proposal assumes an appropriate choice based on the place that the content occupies and the learning expectations within the course programming and the specific conditions of the group of students with which it works.

2. Independent student work.

In this phase, time is offered for individual work, in pairs or in subgroups.

There are several sub-phases in it:

- appropriation of the problem,
- formulation of strategies-hypotheses-procedures,
- problem solving or student research.

This phase is recorded as a "*independent phase*" in that there is no direct teaching intervention, and the students are left to face the problem on their own. There is no significant learning without this stage of confrontation with the problem. When carried out in the classroom, however, an appropriate, precise, and active teaching action is necessary.

In this phase, the person must know some strategy that allows the solution of the problem, but not one that is based on the knowledge that is to be taught. On the other hand, it is appropriate that the problem allow the use of several strategies.

3. Interactive and communicative discussion.

With the teaching guide, this third moment allows spaces for the assessment and contrasting of results, solutions or elaborations provided, argumentation and communication coming into play.

4. Closing or closure

This *closing* or *closure* allows an activity that pedagogically "concludes" the topic, or the contents worked on. It is a fundamental cognitive synthesis for learning. Through this teaching action a "link" is offered with the mathematical knowledge that the professional mathematics community has built. It is important that this closure is not artificial or distant from the recently experienced process.

Closure is about the acquisition and structuring of knowledge (concepts, procedures, methods) that were used throughout the process. It is confronted with known knowledge, although in an accessible way. Here you can influence the strategies if there were several, introduce a critical analysis of the actions carried out and propose complementary activities that strengthen the understanding of the knowledge worked on. It is advisable to reformulate the new knowledge acquired in writing, always with the help of teachers.

Stage 2 (of mobilization and application of the knowledge learned) is about getting some of the learned procedures to work mechanically, which will expand student mastery of the forms of expression or representation of knowledge such as formulas, symbols, graphs, and diagrams. And it also includes the application of new knowledge in different contexts. It is a stage in which connection with other areas and some additional reflections can be made. At this stage, the evaluation of the knowledge learned is proposed. It is important not to incur excessive repetitions and development of activities without interest. They should be tasks to reinforce learned knowledge, since it is always possible to find problems and actions that complement, pointing out underdeveloped aspects or showing motivating ways of applying this knowledge.

The development and combination of these stages 1 and 2 and even the four phases of the first stage are part of the teacher planning. They should not be seen in a linear fashion or as a mandatory sequence. It is appropriate to start a lesson with a problem, but before going to a new topic, it can be started with the mobilization and application of learned knowledge that is judged necessary to advance in the new learning. It will always depend on the nature of the topics and the conditions in the classroom.

This style of organization of the lessons is transversal in the teaching of the mathematical topics that make up the school program. But it is more than that. It can be developed in other subjects. Training in the proposed methodology also promotes the development of capacities to carry out research in other natural or social sciences and in the humanities. This is so because the method followed is similar to the one that dominates in the construction of knowledge, that is, in research: Problem, proposal-hypothesis-conjecture, resolution, contrast of solutions, consignment of results in relation to knowledge.

Style Considerations for Organizing Lessons

In this approach, duly contextualized problems constitute a source from which the learning process starts. These problems serve as situations not only to apply mathematical concepts or procedures but also to build them. What it is about is that the necessary Mathematics in the problem can be developed in addition to the understanding of the actions they perform. An activity is carried out in which one goes from establishing strategies very close to the specific context used towards others of greater generality. When this happens, a model is invoked that can be used in other situations, to solve other problems (different but similar). Higher levels of generalization and abstraction will offer broader mathematical possibilities.

In this style of organization of the lessons there are interactions between students and teachers as well as among students. Mathematical dialogues are presented. It is relevant to provide enough information so that all students have at their disposal the background and the inquiry that the problem poses, to later classify, interpret, and build.

It is the core of a teaching intervention in terms of guidance, advice, and formulation of appropriate questions but with full awareness of the moment in which to act and in which the problem must be confronted. It is not appropriate to offer the answer or the solution route to the problem, since the possibility of activating the cognitive actions that are going to cause learning and development of mathematical abilities is removed. However, in certain cases solutions and answers will have to be offered (when the student cannot act autonomously). What it is about is having a general perspective where it is sought to generate this independent action.

It is advisable that the indications are just those necessary for the activity to be followed. These indications must be adjusted individually. It seeks to create a culture different from the one that is common among students, teachers and parents. Teaching work should not be seen as “solving” problems for each student. This style promotes or “forces” to engage in learning and to have a participatory attitude.

An important issue is that the same problem can serve different purposes depending on student conditions. If the mathematical concepts and procedures involved in the problem are already mastered, it is more of a reinforcement exercise or the simple application of content. Problems in the affirmation of learning set in motion previously achieved learning. That is why a teaching action that accurately identifies the role that a problem can play based on the student context is essential.

The problems that arise must have a certain complexity, if all the means (concepts or procedures) to solve them are not possessed. Precisely in this confrontation, it will be possible to access or elaborate the theoretical resources that provide a solution to the problem posed.

In this style, people are confronted with different types of tasks and explorations, with the teaching action they are motivated to find answers or condense their contributions by carrying out the mathematical processes of *To represent*, *To reason and argue, To Communicate* or *To connect.* From the outset, work is done directly with the approach and resolution of problems, which, with an emphasis on contextualized problems, promotes the identification, use and design of simple mathematical models, that supports the development of mathematical capacities.

This style requires a careful preparation of the lesson, involving the choice of problems, the time to allocate for each step and the teaching action at each moment. This is not only a general guide for the construction of automatic learning but also has a specific central character to social and cognitive interaction in the classroom.

This lesson development assumes:

- specific pedagogical or didactic *methodologies,*

- *management* of the study program (its planning and implementation in precise times and conditions),
- an *assessment* adapted to the style of organization of the lessons.

It has been common to establish school content by dividing each one into small parts for which mechanical and almost meaningless procedures are offered. Teaching in this scheme consists of reproducing these procedures without much understanding of their meaning. In the style proposed here, the idea is different: Problems of a certain complexity are raised and students are not only passive receivers but also active participants in the classroom. Similarly, an activity is promoted in the lessons so that each student contrasts and communicates their ideas and solutions, thus activating relevant mathematical processes.

There must be great flexibility in the use of this style, which will depend on the classroom conditions and context as well as the educational level at which it is taught, but organizing classroom action in this way can offer a general strategy for teaching motivating for the majority of teachers in the country. It is a strategy that, together with continuous professional development and the provision of special materials, would allow progress in significant learning, the enhancement of mathematical capacities and the construction of rich classroom experiences.

Another issue: It is not about using many problems in a lesson, rather a few from which to build the learning in depth.

Finally, it is a style that allows enriching educational work overtime in a precise manner by choosing the best problems, anticipating possible solutions or recurring errors, doing (classroom) research to improve the presentation of problems and the organization of the lesson.

The “oriented inquiry”

In tune with this general style, a strategy for conducting the lesson can be developed through inquiry directed towards the whole class:

- asking appropriate questions about a topic,
- waiting time for answers to be offered,
- reformulation of the questions to advance in the different aspects of the topic, and
- repetition of the process until reaching a cognitive and pedagogical closure of the subject.

This pedagogical methodology requires a very active and attentive teaching role, going through the steps of the suggested style (problem, student work, communication and contrasting answers and closure) but in a more dynamic way, which can reiterate the sequence of steps in a short time. It is a method that can be used at certain moments in the classroom, to be established either by the topic or by the conditions of the class (for example, large classes).

The key elements are the questions must capture student interest, and the sequence must generate increasing variations in the treatment of the topic and in the understanding of the different elements of the topic considered. Questions should always be formulated based on student responses.

General indications

This section tries to offer some general suggestions for the elaboration of pedagogical planning and the development of the lessons.

To integrate abilities

One of the relevant guidelines for the development of classroom action with this curriculum refers to the management of content and specific abilities. These should not be viewed in a disaggregated manner. Abilities are not operational objectives that must necessarily be worked on separately in the classroom. On the contrary, it is appropriate to try to integrate the specific skills in all the learning activities: planning, lesson development and evaluation. Through a single problem it is possible to address several abilities.

Educational timing

Consciously or unconsciously, each teacher adopts an implementation strategy, according to which they must consider several terms (short, medium and long) and the following elements:

- The general conditions where the lesson will take place (socio-educational context, locality, available resources and materials, educational level, number of students, etc.), which intervene in different ways in the construction and development of the lesson.
- The place that each lesson occupies in the development of the curricular aims. The lesson must be understood as included in sequences of lessons on one or several topics, that is, the curricular schedule where the topics and times assigned in the school year are placed and that require a strategic vision.
- A planning of the different moments of the sequence of phases to be developed.

Lesson events

There are several things that are very important to lesson design and are often called *lesson events.*

Table 3. Lesson events

Beginning of the lesson and introduction of content.	The beginning of the lesson is decisive. The problem that will start the lesson must be chosen very carefully, which is associated with the way of introducing the content. The nature of the problems that are most appropriate must be identified based on the mathematical area, the educational level, the topic to be treated, as well as the capacities and processes that are to be favored. It is not only important to select a problem appropriate to the content, but also the way in which it is presented. A written statement is the simplest, but images or technological means can be used.
The problems that arise in the lesson.	It is necessary to identify other problems that are intended to be introduced during the lesson and determine the ends that are sought with them, since they will have a different function from those that serve to start the lesson.
Teaching actions when students work individually, in pairs or in subgroups.	These actions must be foreseen, or at least it is appropriate to design a general attitude. For example, when should suggestions be given and of what type.
Student participation at the blackboard.	Participation at the blackboard in front of the whole group reinforces the competence of mathematical communication, and the security and confidence of each student. However, it must be done with care so as not to offer a wrong view of the content discussed. It is always necessary for each teacher to make the pedagogical closure to show the mathematical content with precision.
End of the lesson.	Also crucial is how the end of the lesson is done. You must decide whether to do a cognitive synthesis at that time or if you wait to do it in another lesson.
The role of cognitive synthesis.	The synthesis or closure of the contents can be done emphasizing its consistency with the elements that were developed in this lesson, underlining their connection with other topics that will be seen in other lessons. This depends on the topic to be taught and its place in the sequence of lessons. This synthesis is essential that it be done either in the same lesson or in one not very distant from the one in which the work was carried out, since the cognitive "mooring" that is required to provoke learning could be lost.
Source: The curriculum authors	

These lesson events must be considered in the pedagogical planning.

Assessment in planning. Also, the characteristics of the methodological approach must be consistent with the way in which student performance is intended to be evaluated. How the topic will be evaluated must be thought from the beginning of planning.

Take diversity into account. As there are different talents and intelligences, it is very important that time is used differently for each subgroup of students or different individuals. This can be done through a specific modulation of the suggestions or indications that you can give. Here at least two dimensions are relevant. the cognitive differences associated with the various approaches that students may have to learn Mathematics, and on the other hand, those related to talent and dedication to study (not everyone has the same talents in Mathematics) or the willingness to spend a lot of time on it. Some people have a better facility for visual representations of mathematical concepts and procedures, others have a greater facility for sequential approximations, number systems, etc. It is not easy to identify these cognitive differences in students, but it is vital to keep the issue in mind, since it can allow a better use of the classroom action for all. Answering this is favored within the lesson when processes related to mathematical objects that have different mathematical representations are selected. The treatment of differences in talents and disposition to study is a more complex matter.

Include conceptual understanding and procedural skills. In pedagogical planning it is essential to reflect on the mechanisms so that the problem allows generating conceptual understanding, as well as procedural mastery. Procedures and conceptual understanding are decisive for the development of mathematical competence. However, the understanding of concepts allows a better assimilation of procedures. In the same way, it is possible to strengthen mathematical competence based on procedures with timely modifications of its different elements.

Interactions in the institution

Although the implementation of the curriculum is carried out individually, it is opportune that several of these dimensions be carried out with the collaboration of more teachers of the institution who teach Mathematics. Pedagogical planning, lesson design, assessment of the results and experiences obtained in the development of the lesson, as well as the incorporation of new knowledge, can be topics for regular sessions in each institution. These activities allow broadening and deepening of the action in the classroom that each teacher carries out. It is a means to carry forward a continuous program of research for classroom action.

IV. METHODOLOGY

In Mathematics Education, methodology and management have made great progress, largely due to its consolidation as a scientific discipline independent of Mathematics and General Pedagogy. Beyond this, many methodological and classroom work strategies have been rethought thanks to digital technologies.

This chapter will offer general indications on various components of the curriculum and classroom action. They are presented in six sections:

- About mathematical areas
- On mathematical processes
- On Student Diversity
- On the use of technologies
- On attitudes and beliefs
- On the use of the History of Mathematics

The extensive collection of tips and advice provided here is a guide and a reservoir of resources. It is not intended to replace professional work in educational task design, nor to offer an inadequately specific level of methodological actions. In some cases, only very general guidelines are given. More specific indications are offered in the study syllabuses, but they should always be seen as suggestions and as an orientation to be used in a flexible and creative way.

About mathematical areas

It is appropriate to start with guidelines on the mathematical areas, since these are the ones that organize the study syllabus. In the first place, it is relevant to visualize the relative place they occupy in the curriculum, and then offer some indications in each one.

The five selected mathematical areas participate with different intensity. The following graph of these study syllabus has been constructed considering the relative places that can be calculated based on the times that are expected to be dedicated to the topics included in them.

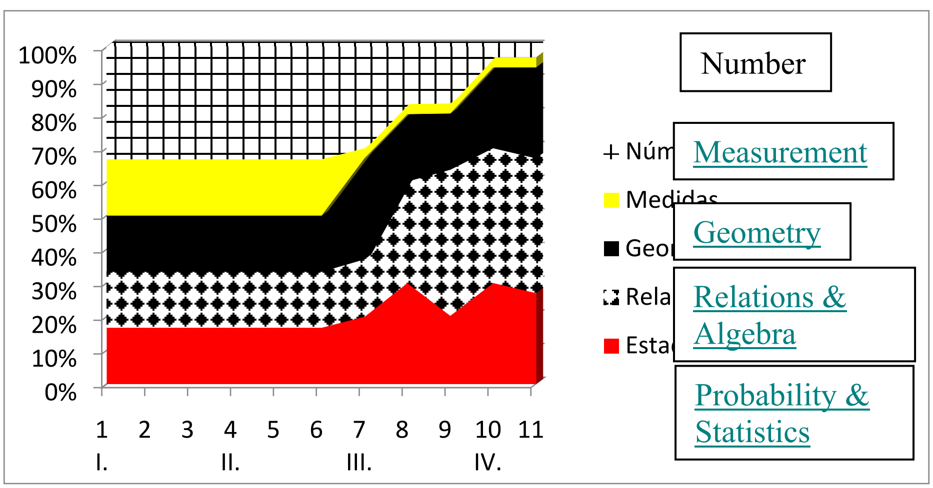

Figure 2. The five mathematical areas in the four educational cycles.

Relations and Algebra is constant in the first two cycles, doubles in the third and has the largest space in the Diversified Cycle (High School). *Geometry* is constant in Primary (grade 1-6) and increases a bit in subsequent cycles. *Number* occupies a very large place in the first two cycles, is relevant in the third and decreases a lot in the last cycle. *Statistics and Probability* is constant in the First and Second cycles, increases in the third and the diversified, without surpassing the place of *Relations and Algebra. Number, Measurements* and *Geometry* occupy 70% of the preparation in Primary.

In addition to an integration between the different educational levels and cycles through common mathematical areas, here are some perspectives that depart to a certain extent from the previous curriculum:

- Integration of the topics of number and operations with an emphasis on performing calculations and strengthening number sense.
- A focus on the area of *Measurements* that seeks to support mathematical learning and establish connections at all educational levels.
- A focus on *Geometry* that includes an emphasis on spatial sense, movement, and the use of coordinates, and a special relationship to Algebra.
- Integration of the topics of algebra, relationships and functions, and their insertion from Primary in a gradual process.
- Enhancement of the place of *Statistics and Probability* from the first cycle to last.

The following provides some general guidance on each area.

Number

A more integrated approach to number, operations and calculations is sought, that is, a special perspective of close connection between operations and numerical representations. In Secondary, sometimes the great numerical sets (**Z** , [redacted], **R**) have been placed in an abstract way, which appeals more to the memorization of properties than to the usefulness of numbers and their operations. It is desired to emphasize a very practical sense of numbers and their properties, especially by solving problems drawn from a context. The use of set theory elements is saved for 10th Year and will be included in the area of *Relations and Algebra,* although it is also useful in *Statistics and Probability.*

It is intended to give greater relevance to the calculations, which allow developing numerical abilities or skills. In this sense, it is proposed to strengthen mental calculation and estimation. This view is associated with problem solving and active contextualization. Mental calculation, for example, can be cultivated from the outset as a special mechanism for mastering numerical properties and as a training for mental skills. However, it is not advisable to fall into the temptation of exaggerating the value of procedures over conceptual understanding. Without procedural mastery, the possibilities to solve problems are affected. At the same time, without a conceptual understanding, procedures are forgotten more quickly, and significant learning is not achieved. Similarly, calculation activities in the classroom allow strengthening the search for different solutions. The registration, explanation, criticism, and communication of computational strategies allow favoring important cognitive processes, which help in the development of mathematical competence.

Number plays a central role in the First and Second cycles, it is notable in the Third cycle, and it is developed transversally to other mathematical areas in the Diversified Cycle. In Grades 7 and 8, integers and rational numbers are introduced, and some value is given to the various decimal representations of rational numbers, opening the way to the introduction of irrational numbers. The concept of irrational is complicated from many points of view (epistemological, cognitive, pedagogical) and its introduction must be done with caution.

A strengthening of *number sense is sought* through an appropriation of the absolute and relative value of numbers. This refers, for example, to the use of numbers to represent dimensions or entities of reality, to the numerical estimation of values and arithmetic operations, to the "reasonableness" of calculations. Number sense is strengthened with a command of operations and the properties they have, for example with the decomposition of numbers using the properties of the positional and decimal system ($15 = 10 + 5$). A number sense allows you to see that a sum as $\frac{10}{11} + \frac{12}{13}$ approximates 2 without having to do the math. Another example: accepting that the average weight of people is 456 kilograms would show a lack of number sense. In the same way, the number sense, closely associated with operations and calculations, is what allows us to decide on the most appropriate strategy to face a problem: mental calculation, approximate estimation, systematic work with paper and pencil, the use of a calculator or even the computer.

One of the central purposes for this area is to promote the multiple representation of numbers such as: 18 = 10 + 8 = 9 + 9, or to understand, for example, that rational numbers can be represented as fractions, decimals, percentages: 1/2, 0.50 and 50%.

It is desired that the properties of certain numbers be distinguished progressively: even, odd, prime, square, etc. As they advance, already in Secondary, they will have to identify and apply the properties of different number systems in an abstract way. For example, that some properties are preserved in some number systems but not in others, as happens with the multiplication of natural numbers, which here is always greater than or equal to the numbers that are multiplied, but this is not necessarily the case when these numbers are not rational (for example numbers between 0 and 1).

Another element that we want to emphasize here from the First cycle is the learning of the relationships between the different operations, which prepares the way for the learning of more abstract properties, which will be studied in Algebra.

The introduction of operations considers cognitive criteria, so division, which is more complex than addition and multiplication, must be properly introduced in different years. In the 1st year emphasis is given to addition and subtraction, in the 2nd to multiplication and in the 3rd division begins. Although a spiral approach is proposed in the introduction and treatment of mathematical topics, it is also sought at times to deal with certain contents with greater breadth and connection, avoiding inadequate repetitions in different school years that do not cause significant learning and often overload some levels with content.

In the curriculum, the Lowest Common Multiple and the Greatest Common Factor were moved to 7th grade, and introduced within elements of number theory to favor a broader treatment.

An important matter in the First cycle is that the treatment of numbers less than 100,000 is now included, to tune in with the claims of a scenario in which the largest numbers occupy an everyday place.

On the other hand, the treatment of fractions and decimals is concentrated in the second cycle. The first cycle is intended for a strong preparation with natural numbers.

In the study plans, the set of natural numbers including zero will be used, that is, the following will be used: [redacted] = {0, 1, 2, 3, 4,…}. This has been traditional in Costa Rica.

Geometry

Geometry is considered as the organizer of the phenomena of space and form, and in particular geometric objects are seen as patterns or models of many phenomena of reality. That is, an approach to Geometry based on the study of ideal and abstract objects is not privileged, but rather one that assumes the geometric relationship with spatial environments. This seeks to strengthen a greater visualization in Geometry by establishing close contacts between visual representations and geometric shapes. In this way, the construction of geometric learning is appealed to in increasing phases that go from the intuitive, manipulable, pictorial, and visual towards the most general and abstract representations. The need to ascend through different levels in geometric learning is reinforced.

This is associated with an approach that seeks to give greater presence to the "spatial sense", that is, the identification, visualization, and manipulation of forms in space. In this way, the sense of figures, bodies and solids starts from the early years, with the physical representations and objects of the environment that can be accompanied using technologies. For example, the representation of three-dimensional figures, their translation, the use of color, textures, sounds and all the possibilities that the multimedia resource can provide, radically new ways to approach the teaching and learning of three-dimensional geometry. This is done gradually in all the syllabuses from the first cycle. It is not appropriate to emphasize the use of formulas but rather the visualization of forms in space.

An introduction to coordinate geometry and analytics appropriate to the different cognitive levels is intended. The analytical geometry present in this area is reduced to the representation in coordinate systems of points and some geometric figures such as the circle. Axial symmetry is studied, which has many interesting examples, and some transformations in the plane (translations and rotations) are introduced in the Diversified Cycle. The introduction of these topics favors the links between Geometry and Algebra, an important dimension of contemporary Mathematics.

It is proposed to introduce the movement of geometric shapes, one of the important topics that developed from the 17th and 18th centuries, opening a new orientation in Geometry (revolutionarily expanding the results of Antiquity). The movement of points and geometric entities allows to build new entities (curves for example) and visualize the usual ones in other ways: a dynamic sense of some geometric properties such as the relative positions and transformations of points and shapes. The treatment of movement in Geometry had been difficult to incorporate into school curriculum due to the limitations for drawing and its graphic presentation. With digital technologies this changed radically. The presence of diverse software for dynamic geometry and geometric representation for many years allows us to approach geometric phenomena including this essential property. But it is more than that technology allows us to rethink the logic of the curriculum and much of its content in Geometry and in other areas. This dynamic sense can be introduced into congruences, similarities, and linear or rotational symmetry of transforming objects, allowing for close connections to *functional thinking.*

A treatment with coordinates that is supported using technologies allows very rich opportunities for the multiple representation of its geometric objects, one of the important characteristics of Mathematics. By means of coordinates, algebraic procedures, objects, and mathematical properties can be represented and manipulated in ways that are very difficult to achieve without coordinates. On the other hand, the current perspectives of Geometry are strongly placed within Algebra and functions, and this allows showing the modern vision of this mathematical discipline, which at the same time will be very useful for many students when pursuing higher studies.

In Primary, it is proposed to work on Geometry through very intuitive and contextualized approaches that must be formalized in Secondary in certain subjects. Synthetic geometry (without coordinates) remains key in terms of generating reasoning and proof capacities. In the first cycles, the recognition of figures and geometric properties stands out. Spatial sense and the study of solid figures in the plane are cultivated. This treatment is deepened in the Third cycle, especially in the 8th Year, where an emphasis is placed on logical and deductive aspects, that is, on reasoning, argumentation, and proof. Under this vision, it is intended to deal with the topics of congruence and similarity of geometric figures, introduced in a simple way through the concept of dilation (which is new), with which a connection with coordinate geometry is again achieved.

In the curriculum, the topics of this area, especially in Secondary, have been selected based on the criteria of pursuing the development of mathematical competence and at the same time providing the contents and instrumental capacities for later professional training. Those contents whose contribution did not favor either of the two were excluded from the proposal.

Trigonometry is introduced in 9th grade favoring the connection with Geometry. It is essential to emphasize the use of models where trigonometry is involved.

Measurement

Measurement is a characteristic of some physical (or mathematical) objects. Not every attribute is quantitatively measurable, and in the case of those that admit measurement there is always a *sense of approximation.* Both for the subject who does it and for the instrument used, an error percentage appears. This is one of the key issues in this area. Measurement is related to number sense, estimation. The same attribute that is common to several objects allows the comparison of measurements, and therefore to appreciate similarities and differences between objects. In the same object, its attributes are likely to have relationships whose study can be done through measurements. For example, a drawn triangle is an object that has perimeter and area, and there is a relationship between the two. And furthermore, when changes are introduced in one attribute, it may be that others remain invariant or change in a certain way. For example, there may be several rectangles with an area of 60 square centimeters but with different perimeters as the width and length values are changed but the area remains the same.

Measurement is proposed as a very rich source to introduce mathematical objects and procedures, to make connections with other mathematical and non-mathematical areas and with many situations in context. Measurement can support the study of various mathematical concepts, such as change and invariance under some transformations. In the same way, the units of measurement can be manipulated as variables (especially when converting units) and therefore be able to motivate a treatment by means of more general algebraic procedures. Topics such as mathematical proportionality or the similarity of figures can be generated using maps, which express relationships of position measurements through various scales. The use of non-linear scales, for example logarithmic, can be used to create real models, in the final levels of Secondary education.

Measurements are associated in the first cycle with the *Number area* and in the second cycle with *Geometry,* but it is also required for some *Statistics and Probability* topics*.* In the Third Cycle and the Diversified Cycle, *Measurement* is introduced transversally to the other mathematical areas. This transversal presence of measurement in Secondary contributes to a contextualized treatment of several mathematical topics and to a sense of reality that school Mathematics must possess.

Here measurement has been incorporated relating to computing dimensions such as storage capacity and data transmission speed, which is in tune with the computerized and technology-rich environment in which we live.

Relations and Algebra

There are several relevant indications in this area. Functions, which in the past have usually had only a very abstract treatment of relationships between elements of sets (correspondences, domain, range, codomain, etc.), are placed here in another more concrete perspective: exchange relationships between 2 variables (that depend on each other). Functions seen in this way are associated with more general relations, such as relations of order (less than or greater than) or relations of divisibility, etc. Issues such as proportionality, percentages, rates of change are part of this area.

The concept of change or variation, which is also common to data analysis, forms a central part of the topics in this area. It could be said that change processes can be modeled by mathematical relationships and functions, and these can have different representations: graphic, tabular, symbolic.

Another important issue is a "functional" treatment of the manipulation of symbolic expressions is favored, for example equations, factorization, and simplification, which allows giving meaning to various topics of this type and starting training in this functional thinking from the beginning in Primary although gradually.

The association between functions and algebra allows coherence to be given to much content that is often dispersed in the usual curricula.

The topics in this area, for example in the Diversified Cycle, have a close connection and continuity with professional training in many careers that require Mathematics. For this reason, in the last years of Secondary Education, content and skills of the functions are emphasized, with a strong orientation towards problem solving and modelling. For example, transcendental functions (exponential, logarithmic) are treated with this exciting new view. The learning and use of functions in different contexts completes the mathematical preparation provided by this area.

The use of algebraic computing technologies has been considered relevant in topics such as factoring or handling polynomials, since they give natural space to the participation of new software or hardware systems, and this is then recorded in the rethinking of some of the curricular contents and above all the approach with which they are introduced.

Algebraic topics are integrated and concentrated. For example, the study of the linear (equations and functions) is concentrated in the 8th year and the quadratic (equations and functions) in the 9th year. This prepares the introduction in grade 10 of the abstract study of functions. With this vision, which has permeated the programs with relationships and functions since the First cycle and which is deepened in the Third cycle, it is easier to learn the most abstract aspects of the functions.

This mathematical area, in connection with other areas in this curriculum and with a good preparation from Primary, becomes the heart of Secondary education.

Statistics and probability

Statistics and Probability takes on much greater prominence in this syllabus than in previous ones. Before, when it disappeared in the Diversified Cycle and by not including applied or statistical probability, leaving it outside of the Baccalaureate national examinations, powerful means for understanding and organizing information were removed. Since the 1990s, its use has become widespread and its place in the curricula of different countries has been enhanced due to its notable presence in everyday life. This is an area that allows us to better visualize the role of Mathematics and contribute to positive attitudes and beliefs around this discipline. That is why this area has a strategic place, which directly feeds the sense of mathematical competence around the description of reality and the cultivation of problem solving in diverse contexts.

The addition of more probability topics in this program seeks to develop thinking about randomness, and abilities to deal with chance, the unpredictable, and uncertainty, characteristics that participate in knowledge and in life in multiple ways. Probability connects significantly with *Number* and *Geometry and* must be treated informally in the early years and then advanced in abstraction in high school.

The relevant place given to this area is due to the role played by information and the handling of chance in modern society. In the 21st century, people capable of understanding, interpreting, and using information to understand reality, solve different problems and make intelligent decisions are required. The topics of Statistics and Probability are every day a requirement to be able to understand what is happening in the world and to be able to act. The topics of this area are introduced in a gradual, intuitive, and practical way from the first cycle and preserving that tone throughout Primary education. Already in the Third cycle, the concepts and techniques introduced are reviewed and formalized, which are summarized and completed in the Diversified Cycle. At all times, which is natural in this area, the themes are presented through real problems, which generate connections with other subjects such as Science and Social Studies, and which allow positive attitudes about Mathematics to be fostered.

Some considerations on this area are illuminating. On the one hand, one of the fundamental issues that is persistently developed is that of data variability. It is very important to insist that the representation and modeling of many phenomena is done through data, and that the different data sets can be compared and thus provide more knowledge of the starting phenomena. Similarly, a data set requires instruments for its description (mean, median, mode, range, deviation). Its teaching must be done largely based on its application in the analysis of information and resolution of problems and not as objects in themselves. This is relevant, because sometimes school statistics are mistakenly seen as collections of formulas and a mechanical handling of these instruments.

The representations of statistical phenomena must go from simple to more complex forms at different levels of schooling, always from the most intuitive in the early years to the most technical in later years, without forgetting that the same set of data allows different representations to be made.

A substantial detail to consider in the teaching of Probability, especially in Elementary School, is the teaching of the difference between the numbers that represent data values and those values of frequency with which those first values occur. At the heart of this discipline is the distinction between, for example, that it cannot be predicted that a head or tail will be obtained when a coin is tossed, while the trend of a large number of tosses can be determined.

A close relationship with the use of some digital technologies is suggested since much of the data can and should be "processed" by means of such instruments. This is relevant because there is a vigorous increase in technological instruments for data analysis and mathematical modeling. The Internet today provides access to a variety of data that can be explored in the Statistics class.

A Costa Rican high school graduate should be able to compare and judge the validity of arguments based on data in everyday life, identify common errors and distortions in the media, discover the rationality of statements about the probability of events, as well how to handle the basic ideas of sampling and perform simple applied statistics. Like reading and writing, the management of Arithmetic, Geometry, Algebra, and other forms of Mathematics have been part of citizenship literacy for ages. Statistics and Probability must be conceived as part of citizen literacy in the current historical scenario.

On mathematical processes

Consistent with the theoretical foundations of this curriculum, once some method guidelines have been offered on the knowledge and skills that organize the syllabus, it is relevant to provide suggestions on mathematical “processes”, that is, ways of understanding, learning and using the knowledge that they promote transversal cognitive capacities and mathematical competence.

Indications for each process

To reason and argue

The process is activated in all areas in multiple ways, for example in the study of regularities and patterns, in the justification of the congruence of triangles, the choice of a mathematical representation and its manipulation, in the solution of equations, among others. Justification and proof are an essential part of mathematical tasks and therefore must occupy a special place in school education.

A relevant place is occupied by the action of conjecture since it is a central path for discovery. In general, it is about raising a conjecture and looking for the means to justify it (in adaptation to each educational level), either by means of specific materials, diagrams, calculators, or other instruments. Conjectures will have to be made on more general or abstract topics as school progresses and increasingly the more precise or technical mathematical forms will have to be used. The argumentation must also be cultivated in a gradual way, first resorting to oral forms, then written and later symbolic. Likewise, the forms of reasoning by contradiction, induction, use of counterexamples and the different forms of deduction must be introduced little by little.

This process can be reinforced through group activity, in which the arguments or justifications provided by each student are contrasted, always with the teaching guide. In the same way, the errors that are made are very useful opportunities to improve mathematical reasoning processes and to advance the associated general mathematical competence.

To pose and solve problems

There are some elements that are worth underlining. In the first place, not every problem can lead to mathematical ideas even if it is interesting or fun, so the teaching action is decisive for the design of appropriate problems. Second, in each mathematical area it is possible to carry out this process in different ways, but always gradually. Problem-solving strategies must be introduced not in an abstract way but in specific instances in the chosen problems. Sometimes it will be to promote the use of diagrams, other times pattern recognition, or proof with the display of cases, etc. Similarly, it is necessary to prepare students in the different stages of problem solving, such as understanding them, drawing up action plans, and evaluating or monitoring actions.

The use of models, on the other hand, must be done in a staggered manner with the teaching of the various strategies. Modeling activities can only happen with active student engagement which is vital for contextualization to be successful in teaching. Modeling is an action that develops in a natural and privileged way when it is part of an educational framework where the organization of lessons through problems is central.

To communicate

This process is associated with an essential characteristic of mathematical endeavors: To be "correct" a mathematical idea must be accepted by a professional community of mathematicians. There are specific rules for doing this, which is important to include in school programs. The process suggests communication at different levels and forms, from the simplest such as oral or written, to graphic, symbolic and formal.

It should be noted that not every topic lends itself to rich communication activities. For example, algorithms are in this sense less useful than concepts, however through an appropriate teaching action it is possible to trigger mathematical communication. Communication and mathematical thinking, particularly argumentation, are intertwined in mathematical endeavors. Communicating requires precise thought.

In the classroom, for example, mathematical communication can be used to introduce new concepts (asking for the elaboration of diagrams, expression of ideas, placement of symbols and expressions), and also to solidify the student's own thinking about the ideas that are introduced in the class. The development of this process allows knowing other points of view that can show different aspects of a mathematical entity. And in the same way, in the activities associated with the “To Communicate” process, it is possible to widen criticality through the natural rational questioning of the assertions and arguments expressed.

The specific mathematical language -sometimes abstract and very technical- is the vehicle through which mathematical communications travel, and for this it must be developed. There is a similarity between mathematical communication and that which is carried out in other areas. Constant practice and teaching guidance are necessary. But precisely because of its technical nature and abstraction, it is necessary to carry out this process gradually at all educational levels.

To connect

It is necessary to have a broad vision of what this process implies in the educational environment. The connections can be developed in many contexts: for example, within each mathematical area (such as when applying the procedures and operations of natural numbers to rational or real numbers). But also, between the different mathematical areas and in general with other subjects. Mathematics, by its very nature, has the potential to support transdisciplinary processes that must be cultivated from the earliest school years. Knowledge must be visualized as an interconnected reality full of links.

This type of school education allows for a deeper and more precise understanding of mathematical objects, but also allows the cultivation of student abstraction, since the generalization and universalization of methods and ideas forces greater abstractions. Observing the applicability and interconnectedness of Mathematics reinforces their appreciation and enjoyment.

However, it is not so easy to introduce connections in the classroom. Mastery of the different mathematical areas is required as well as some precise knowledge to support these connections with special problems. It is important to plan ahead for the introduction into the classroom of the connections of the topics of a lesson.

To represent

The representation and manipulation of mathematical objects should not be seen as an end in itself. It should be understood that these representations and their laws express both mental actions and characteristics of mathematical objects. *Representing* must be closely linked to *communicating, reasoning and arguing* and *posing and solving problems.* Otherwise, its meaning is distorted towards a merely mechanical use, without really being able to achieve understanding.

Mathematical representations by symbols, expressions, diagrams, graphs or by technological means are historically elaborated products, so they change and require teaching and learning actions. While unconventional or even intuitive and personal ways of representing mathematical ideas are possible early in school, it is important that more conventional and technical ways are gradually taught. The learning of formal mathematical representations allows communication to be carried out, since it offers the language and objects to understand each other. If everyone worked with their own individual representations, there would be no room for communication.

It is highly relevant that mathematical representations of ideas and objects can easily obscure the complexity of the ideas and objects they represent. For example, when x is written to represent a variable very simply and usefully, one can lose sight of the fact that the mathematical meaning of what a variable is can be complex and difficult to understand (and requires educational actions for its learning). The same thing happens with the decimal positional system, which allows us an easy use in carrying out arithmetic operations that can hide the complexity of the mental and mathematical actions that it represents.

The cultivation of diverse representations allows a better organization of mathematical ideas, to advance their understanding and the development of new mathematical forms. This is the same thing that has happened in more general mathematical matters not associated with education. Without the symbolic and graphic representations that mathematicians built, the progress of new stages in Mathematics would not have been possible.

It is essential to insist on diverse representations for mathematical objects. It is about showing from the simplicity of expressing a number (5+3, 4+4, 8) or expression [$3x+3$, $3(x+1)$] in different ways, to other more complex and abstract representations. Each representation of a mathematical object can reveal an aspect or property in a special way. For example, $\frac{3}{2}$ expresses a number by means of an operation or a relation, which the same number 1.5 does not do. Choosing the appropriate mathematical representation to solve a problem or build a model is one of the most relevant topics in the teaching of Mathematics, a practice that has been favored using digital technologies.

The mathematical representation incorporates, at the same time, an abstraction of properties, allowing their effective manipulation either to build new concepts or theories, to solve problems, build models or to express more complex entities in different contexts.

It is very important that the abstraction of mathematical representations can be advanced step by step in order to enhance the set of Mathematics that can be learned and used. With the progress of different forms of representation, with more and more abstraction, there are more opportunities to build more interesting and complex models in different situations.

Other suggestions about processes

Adaptation to the educational level and the mathematical area. Our processes must be identified and adapted appropriately at each educational level. In addition, their participation is different in each of the mathematical areas. *To communicate* in number, operations and calculations is perhaps easier to do in Primary education than in other areas. In some school years certain topics and areas favor one process more than another. For example, in 8th grade congruence and similarity of figures (in Geometry) or the introduction of irrationals in 9th grade (in *Number)* promote the process of *To reason and argue*. *To connect* to the environment and other subjects are easy to make in *Statistics and Probability* at all times, and connections between Geometry and Algebra are always favored. Deliberate teaching action is what encourages a process to be activated and, therefore, pedagogical planning and the design of mathematical tasks must be carried out carefully.

Design special problems. There are richer special problems that stimulate more processes than others. A problem should not be designed "exclusively process-oriented", but rather they should emerge from problems oriented to learning abilities.

Enrich problems to achieve greater process activation. The same problem can be modified in its didactic variables to make it richer. Enriching problems is one of the most important tasks in the teaching of Mathematics.

Implement various mathematical processes in a problem. It will not always be possible to develop all five core processes in one lesson. In some lessons, some processes, or others, or only one, may be proposed, but it is important to take this into account when planning and developing the lesson. When a contextualized problem is chosen as the generating center of a lesson, *To pose and solve problems* are called upon, but it is also possible to activate *To reason and argue*, *To connect* and *To communicate* there. Not every problem lends itself to this, but tasks must be designed where it is possible to activate these processes. An advantage of organizing the lesson through context problems is the possibility that almost all mathematical processes come into play. But it all depends on how you plan and how the lesson unfolds.

Promote the writing and communication of responses. A privileged activity that summons these last mathematical processes is the careful writing of the solutions and their oral or written communication in the subgroup or in the whole class.

On Student Diversity

Educational circumstances in Costa Rica are not the same in all institutions and regions. There are differences between urban and rural, between areas of greater socioeconomic development and marginal urban areas. Added to this diversity of realities that generates different levels of school achievement is the diversity of individual conditions (from cognitive and personal to cultural) in relation to learning.

In some way, adequate opportunities must be offered to all and fulfill the purpose of national education that promotes an inclusive and democratic perspective. If curricula are restricted or minimized to localities or social sectors for reasons of socioeconomic or geographic condition, social inequalities deepen. The idea here has been to propose a basic general curriculum for all with the necessary and sufficient content to generate knowledge, and above all, the mathematical capacities required by the context in which we live.

It is essential to understand, however, that the actions to address diversity are in the hands of teachers and educational authorities, and not in the curriculum. What the curriculum can do is offer some general means that can be adopted.

An inclusive approach in the classroom is necessary. It is important to offer options to the various segments of students at different educational levels. At the international level, the options for channeling alternatives to student sectors are very varied. In some cases, it is done through performance segregations that can even start in Primary. In the Costa Rican context, apart from material and logistical factors, and in particular for cultural and historical reasons, it would not be possible to introduce segmentation in education. Apart from that, there are underlying pedagogical and educational reasons why an approach that makes segregations is not appropriate, and rather an inclusive approach that fosters collaborative attitudes in classroom action is necessary.

Consider students that lag behind. In an inclusive approach it is necessary to give particular attention to lagging students, refining the mathematical tasks that are proposed and designing the specific teaching intervention in the different phases of the lesson. Lagging behind can be the result of many circumstances and can be overcome if given proper attention. It is necessary that this segment of the student population be considered in the planning of their lesson and in general in their educational action, in order to develop the learning goals that are proposed. In cases of “curricular adjustments” according to official regulations, it is the teacher's responsibility to comply with the indications that emanate from the legal framework that governs these situations.

The treatment of complexity serves to address diversity. One way to respond to the different needs and qualities of the student population is through an adequate treatment of the complexity of mathematical problems in the classroom. The matter becomes methodological. It is important to identify the aptitudes and the disposition towards Mathematics and to offer appropriate classroom actions to the different needs. Teachers with a lot of experience already tend to do this on a regular basis. It would be about establishing actions with different weight in the levels of complexity of the problems. For some people a greater weight in problems of reproduction and connection, for others a greater weight in those of connection and reflection, and for the majority probably a distribution that is “balanced”. It is a decision that must rest in the hands of the teachers. This may also lead to additional extra class actions.

There should be special attention for students with greater talent or disposition towards Mathematics. There has been insufficient support for students with talent or with a greater disposition towards learning this subject. Actions such as Mathematical Olympiads or programs in the Diversified Cycle such as MATEM (organized by public universities) are useful in this direction, but they are not enough, and this segment of students must also receive specific attention.

This can be done with more in-depth Mathematics, which can also impact the class and learning in a positive way. Not only is a fair response given to the learning needs of these people, but their action can be introduced in the classroom in a collaborative way to benefit all student segments. Those who are more talented or more willing to study can support and help students with less progress in learning and this is associated with general values that education should promote, such as equity, inclusiveness, and solidarity.

On the use of technologies

The use of technologies is central to enrich and re-dimension problem solving and educational strategies. In these study plans they are incorporated through the treatment of various topics, increasing their use with the advancement in the school years. This is done through timely indications, as well as others that are placed at the end of each area in each cycle. The selection of some content assumes this approach (such as rotations in the Diversified Cycle). However, the specific abilities that are included are relatively few. This is so precisely because the country does not have all the preparation conditions for a more intense introduction. The way it is placed in the syllabus, however, allows the technologies to be used in a variety of conditions. Over time, the use of technologies should be intensified.

Identify the pedagogical sense, and do not use technology for the technology itself. The use of technology in the classroom must be done appropriately. There are differences in the purposes and possibilities of each technology. It is necessary to be very clear that the use of technologies must be made strictly based on the contribution it offers to the achievement of the stated learning purposes. Its use should not be adopted for the intrinsic value of the technology, whatever it may be.

The calculator must be an auxiliary. The use of calculators should be insisted on from Primary to corroborate operations (mental calculation, estimation) and as an aid in solving problems and contextualized situations.

Use the computer to visualize and experience Mathematics. The computer is a very powerful resource to use in the teaching of Mathematics, if it responds to pedagogical purposes. The so-called dynamic instruments, for example, are especially relevant: Geometer's Sketchpad, Cabri, Fathom or Geogebra type, since they tend to reduce the boundaries between students and those who have developed the activities. Some instruments (such as the indicated software, where open activities are stated that do not end) are closer to the student's point of view than to the teacher's (such as Java Sketchpad). They are instruments to facilitate computations, to support the visualization of mathematical entities and relationships, to favor mathematical experimentation, orchestrate communications, form networks, and mathematize the external reality.

Use some special software programs. In the current historical moment, the access of people to a computer is wide in the country, both in the external social environment and internal to the educational entity. The use of three types of software programs is proposed here in an appropriate manner for educational levels:

- Dynamic geometry.
- Calculation and graphic representation (CAS).
- Simulation of dynamic statistical experiments.

In all of them there is excellent free software -easy to use- that can allow the development of various mathematical topics with greater ease, projection, and experimentation.

The role of the Internet is important. It offers Mathematics and its teaching multiple links with the student environment and with the main social, cultural and educational trends on the planet. Dimensions included:

- Inquiry, evaluation, and selection of pertinent information for mathematical topics. For example, web pages with information on mathematical situations, censuses, Google maps, figures, etc.
- Reinforcement of mathematics learning through specialized sites with interactive platforms.
- Interactive and collaborative learning in virtual educational networks, also through special platforms.

Multimedia is very useful. Audiovisual media are today an extraordinary instrument to generate motivation in the classroom, to record steps of the lesson and incorporate elements of the environment. In particular, both to promote the treatment of topics and to review or analyze educational processes in the classroom, videos have become a very useful instrument.

The evaluation of the use of technologies. It is proposed that the use of technology be evaluated through the problems or exercises proposed, where its use represents an appropriate component. For example, if in a problem the use of the calculator is significant for its treatment or solution (very large calculations that would take a lot of time without a calculator, or the value of a function at a point that is necessary for the exercise), that element should be considered. So, it is not a question of evaluating the technological manipulation in itself but in terms of appropriate problems.

On attitudes and beliefs

It is essential to take advantage of the opportunities available in the classroom to ingrain the attitudes that have been selected as central. This is transcendental since there may be an intimate connection between attitude and performance in Mathematics if the negative feelings that often exist among students and parents about Mathematics are reduced. Keeping it in mind does not mean that in each topic it is possible or appropriate to introduce it as an objective to be developed, since it can distort the meaning and success of the lesson. The promotion of positive attitudes towards Mathematics derives in this curriculum from several global orientations. For example, the choice of the articulating axes works in that direction, appealing to student interest and involvement in learning. Another is that the correct use of technologies is a powerful motivational resource, because in addition to being associated with the current reality of our youth, immersed in a world full of technology, active and interactive dynamics are reinforced that can facilitate student attention.

Here are some ideas to strengthen positive attitudes and beliefs about Mathematics.

Pay attention to beliefs because they are the basis of attitudes. Most attitudes are associated with beliefs that are sometimes very difficult to change, because they may be embedded in the local culture. But if it is known what those beliefs are and pedagogical support is available, it is possible to cultivate new beliefs in people and generate positive attitudes associated with them. When someone thinks that spending more than 10 minutes on a problem is wasting time, thinking that it is poorly formulated or because it is for geniuses and they cannot do it, they are facing a typical belief in the local culture. It is a belief according to which the mastery of Mathematics requires a talent that is brought at birth, and that if you do not have it, then “there is nothing to do”. Faced with this false belief, it is necessary to cultivate the one that affirms that mastery of Mathematics is achieved with work and perseverance. By working with a problem for more than 10 minutes, half an hour or an hour, even if the solution does not come out, learning is achieved, because the mind is developed, theory is reviewed, failed alternatives are explored and the limits of methods are learned. This is fundamental, since precisely the problems that allow fundamental mathematical processes to be carried out and mathematical capacities to arise require dedication, time, effort, and perseverance. If you are only willing to make minimal efforts, you will hardly be able to go beyond problems of the reproduction level and with this, the progress of your mathematical competence will be severely limited.

Involve family members. Especially in the first two cycles, it is essential to involve family members in the development of positive attitudes and beliefs, in order to encourage the child’s mathematical development, provide an environment rich in language, stimulate exploration and value originality. You need to broach the subject with them explicitly.

Make a diagnosis and keep a written record of attitudes and beliefs. Just as a record of school performance is kept, one should be kept of the attitudes and beliefs that are perceived in the classroom. It is a very relevant task that must be adapted to each educational cycle. In the first two cycles, information can be obtained through some specific questions: Do you like to count? What do you think Mathematics is? At other levels, an activity can be carried out on the use of Mathematics in society (a historical circumstance or a newspaper clipping with a news item on Mathematics, for example), allowing this information to be obtained.

A small, carefully crafted survey can also be done. In the third cycle, this record of attitudes and beliefs is more useful, since it is a school context with populations that are generally very heterogeneous, since they come from different schools. Once those who express negative attitudes or beliefs, which are often associated with poor performance, have been identified, they should be encouraged to reach the desired levels of participation, trust, respect, self-esteem, and perseverance. What is desirable is that this type of negative attitudes or beliefs do not go unnoticed, and actions can be attempted to reverse them.

Time planning is required. Both to enjoy Mathematics and to have a better understanding of its fundamental ideas and procedures, it is necessary to have enough time to test, experiment, build, make mistakes, reflect. This implies that there must be a very methodical planning of the time in the lessons.

The language used is important. You must pay attention to phrases like “how ugly Mathematics is”, “this is never going to work out for me”, “I’ll never be able to”, “Math is too difficult”, “Mathematics is useless”, “Mathematics has always been difficult for me”, etc. It is not appropriate to express phrases such as “this topic is very complex”, “this grade’s Mathematics is very difficult”, “here almost nobody is going to pass”. The language used is key to appreciate and enjoy Mathematics, to prevent negative feelings and to enhance self-esteem.

Monitor individual performance. One of the strongest triggers for negative attitudes is difficulty with homework or math problems. It is appropriate to monitor the work of each student and intervene in a timely manner when you observe that someone is having difficulties in advancing with the proposed activities.

Teacher-student interaction is essential. It is important to have an active attitude in the classroom, paying special attention to students in difficulty and to those who have talent. You can inquiry correctly to find out if that person is really learning the concepts. Each student must feel that her contributions are necessary for the development of the lessons. It is essential that the different approaches students provide be carefully assessed to detect those elements that can be recognized and used to strengthen self-esteem.

Promote connections with other subjects. The multiple connections that Mathematics has with the different disciplines and areas of knowledge is another element that must be used to show the usefulness of Mathematics. In primary school there are good opportunities to do this because in Costa Rica one person is responsible for teaching almost all subjects. In High School teachers can coordinate with colleagues from other subjects.

Table 4. Some indications for attitudes-beliefs

Attitude	Indication
Perseverance	*To handle errors appropriately.* If a person makes a mistake, he/she should not be allowed to give up. It should be insisted that he/she continues looking for strategies that allow him/her to find the answer to face and solve problems. Errors should be seen as opportunities to review theory, explore different approaches, and improve reasoning processes. The idea should be conveyed that making a mistake is not wasting time, but rather learning about a solution that was not the right one for that situation. Being wrong is also learning.
Confidence in the usefulness of Mathematics	*To use the environment.* The use of elements of the geographical area in the environment favors the perception of the usefulness of Mathematics to solve problems.
	To emphasize that Mathematics is used to solve all kinds of problems. From the first years of primary school, it is essential to explain the fundamental role that Mathematics plays in solving a large number of problems. Mathematics should be viewed as something practical and that its learning is very important.

Active and collaborative participation	*Always to provide a playful space.* Games create a natural environment where you collaborate and share collectively. It is a very useful methodology to involve the participation and enjoyment of Mathematics. Games of the competition type, such as "Torch" or "Jousting of Wisdom", the Rubik's cube, four in a row, naval battle, chess and others, favor the development of attitudes of healthy competition and enjoyment of Mathematics.
	Group work. Through group work, active participation can be encouraged. This consumes more time for the lessons, but well carried out it allows an adequate appropriation of the learning. These types of activities should conclude with oral presentations. In this way everyone is involved, and other mathematical processes are activated. These presentations allow challenges, force refinement and reinforce arguments.
Self-esteem in relation to the mastery of Mathematics	*To pay attention to negative comments.* It is important to prevent derogatory comments in the classroom about a person's contributions when they make mistakes.
	To use positively the different ways of reasoning and approaching problems. The different ways of learning can be identified in the classroom and thus promote the search for multiple forms of reasoning and approaches to problems.
	Modulate the demands. Activities should be appropriate to the student's emotional and cognitive level. This is a must.
Respect, Appreciation and Enjoyment of Mathematics	*Use of mathematics applications and use of history.* Even if they are only not so elaborated examples of applications of Mathematics, news from the press or anecdotes about the history of Mathematics, these become powerful instruments to cultivate respect and appreciation for Mathematics.

On the use of the History of Mathematics

The role of the History of Mathematics should be conceived as a resource to provide special teaching opportunities, to offer inputs to the logic of the lesson and to generate positive attitudes and beliefs about Mathematics. Some of its uses can be cited:

To show different ways of thinking and mathematical action. For example, when comparing the deductive scheme in Euclid's *Elements* with the use of intuitive and heuristic devices as in The Archimedean *Method (where he uses means of physics and engineering).* Here would be an opportunity to show the sense of what is and what is not a mathematical proof.

Strengthening of the connections between the different mathematical areas. By studying sociocultural contexts of mathematical concepts and procedures it is often possible to find many connections between mathematical areas, for example, analytic geometry in Descartes and Fermat.

Use of history favors connections between Mathematics, Mathematics Education and general conceptions of students and teachers. The History of Mathematics is the best means to establish links between Mathematics, Mathematics Education, and the global culture of a society, and therefore with the general conceptions of individuals. This is very important to build bridges between Mathematics teaching and students.

Enrichment of problem solving. Proposing a mathematical problem from a historical period not only offers the opportunity to identify those relationships between Mathematics and other sciences or cultural dimensions, but also to use interesting challenges that can set processes in motion. There is an exciting intersection between using history and problem solving.

Strengthening of active contextualization. It offers opportunities to develop connections in a natural, realistic way, which is sometimes attempted in an artificial way that does not spark interest. Put another way, History is an instrument to enhance active contextualization.

Strengthening of multiculturalism. Different cultural approaches to mathematical concepts can be introduced by placing them in historical contexts. Contemporary Mathematics tends to be viewed as a Western product, History can allow us to identify the contributions of different civilizations in mathematical tasks (China, India, the Mayans), and therefore cultivate a broader vision of sciences and culture.

To attend groups with socio-cultural particularities. History is a powerful vehicle for introducing precise, local cultural circumstances that can be used for teaching and learning in rural or ethnically or culturally distinctive settings. There are many references to teaching situations in specific cultural or social settings. For example, in Polynesian languages, the notion of distance is associated with the time it takes to reach a place and not with the linear notion that is used in the West, changing the meaning of measurement and the ways of approaching Geometry. Culture conditions mathematical learning.

To serve talented students. It is a privileged medium to serve talented students, who can enjoy the evolution of mathematical topics where procedural ability, intellectual acumen, aesthetics, or sociocultural variables have intervened.

Connections between mathematics and other disciplines: interdisciplinarity. This is based precisely on the fact that in sociohistorical contexts there are multiple connections between scientific and cultural practices. A special relationship that can be introduced from history is the one between Mathematics and literary texts. This can be achieved in a very practical way. Reading original texts and their translation into mathematical language and vice versa. Writing mathematical realities in natural language. Doing this favors communication and other skills that are not necessarily mathematical but are relevant to learning, such as reading, searching for references, searching for documentation, etc. Reading literary texts that appeal to a historical moment can be an excellent way to introduce the nature and boundaries of the mathematical objects of that moment. These may be problems starting a lesson. Art in general (plastic, dramatic, literary, musical) is a valuable methodological resource that can be used very well within historical situations related to Mathematics.

To support the development of positive attitudes and beliefs about Mathematics. By conceiving mathematical constructions as dynamic activities, it is possible to motivate the development of attitudes such as, for example, persistence, which can be nurtured by understanding the necessary efforts over time to achieve the development of Mathematics. In the same direction, greater self-esteem can be motivated, since the history of errors, failures, and misunderstandings in the history of Mathematics is one of its characteristics, as well as of all human intellectual construction. Unsolved or solved with great difficulty, wrong proofs, and various problem solutions can be used to show the difficulties of mathematical construction. There are many recreational problems raised in different historical moments that can promote the enjoyment of Mathematics. At the same time, reviewing mathematical results that have been present in technological progress would contribute to positive attitudes towards Mathematics.

It can be shown, for example, that much of the computer world has its origins in Mathematics, that the modeling of the universe has been done with mathematics, that there is mathematics in the design and construction of bridges and buildings, that aerospace ships have in its mathematical entrails. This fosters respect, prestige, and enjoyment of Mathematics.

There are several didactic options for the use of History in Mathematics Education:

- As a reservoir of anecdotes to motivate and raise awareness. An anecdote can be the reference that allows a subject to remember a mathematical object or result.
- Description of mathematical situations, which situate a context and individual and sociocultural circumstances.
- To determine the sequence or logic of the presentation of some topics since historical logic can suggest similar paths in learning.
- Use of primary sources, problems or texts of mathematicians that can allow the treatment of certain topics with the theoretical tools that were available at the historical moment.

As for the first two, there are not many difficulties in understanding their usefulness. The third use may seem more sophisticated. It refers to a topic that has been raised for many years: the relationship between the history of ideas and learning about them. Undoubtedly, in terms of education, the great transformations of scientific and mathematical ideas should always be kept in mind for their insertion in pedagogical action. That is why historical logic provides opportunities for the treatment of topics, such as when it is decided to start with logarithms and then move on to exponentials due to the way this has been developed in history. The fourth use that is consigned is the design of problems based on primary sources or that appeared as they are in their historical moment. They are extraordinarily useful because they provide both the context and the cognitive frontiers, they allow comparisons to be made that clarify general ideas and the effectiveness of mathematical methods in the light of the present. For example, there have been conceptions about the rigor and meaning of proof, or prejudices such as the idea that only circles and lines should be used in geometric constructions because they are perfect figures.

Placing problems as they appeared in history and as faced by scientists and mathematicians of a time (with specific means) can be a special contextualized resource to carry out fundamental mathematical processes, to develop skills and to put mathematical tasks in the right perspective.

Closely linked to the above, in a historical circumstance it is possible to identify obstacles or epistemological difficulties that could have a parallel with those that could be found in learning (for example: managing the infinite, the immeasurable, etc.).

For these different teaching options, documents or primary sources can be used (texts by mathematicians, Euclid, Diophantus, Descartes, Fermat, Gauss, among others), History of Mathematics books that narrate or recount phases or areas of Mathematics, characters, moments or books that contain guidelines for classroom action. The latter are more difficult to obtain, but in recent years the Internet pages with uses and teaching resources of the History of Mathematics have increased considerably. Here there is a reciprocal confluence. With the use of information technologies, the use of History is enhanced and with it, it helps to configure some pertinent uses of those.

To insist on the character of Mathematics as constructions. It is a deep-rooted belief that Mathematics is exact, and its truths are placed in a “Platonic” world that only the best minds can access and describe. That is false. All Mathematics has been the product of human and social constructions in contexts with non-universal and modifiable rules. In most cases there have been errors and different approximations until reaching what now exists. Mathematics is not so far removed from other natural sciences. And this is important to express in the classroom.

Various strategies for using History of Mathematics:

- Making posters about mathematicians or mathematical results. For example, figurative numbers, interesting proofs such as the one attributed to Thales of Miletus on the right angle of a triangle with a vertex in the circumference and having the diameter as the opposite side, the calculation of the measure of the earth's diameter by Eratosthenes, examples of the different mathematical notations, among others,
- Extra class projects. They can be about the history of π(pi) or Fermat's last conjecture, about measuring instruments, about the history of the metric system, about the origins of Probability, etc.,
- Dramatized recreation. For example, a mathematical discussion (on the use of circles versus ellipses in cosmology, on Ptolemy, Hypatia, or Kepler),
- Videos on scientific and mathematical topics. (on Copernicus or Galileo on heliocentrism),
- Multicultural comparison. For example, several tests on the same result (it could be the Pythagorean theorem in Euclid or in the Chinese civilization),
- Translation of passages from mathematical texts into modern language, etc.

A relevant pedagogical strategy: identify interesting problems (in primary or secondary sources) to solve using the knowledge of the time. This can be done in several steps, which are listed in the following table.

Table 5. Use of original historical problems.

First step	Locate the social, geographical, and cultural context. • Charts, passages from historical books, movies, etc. can be used.
Second step	Raise the problem and its meaning in the Mathematics of that time. • It can be started having students working simple procedures related to the problem. • Identifying the problem in precise mathematical form.
Third step	To solve the problem • Through the heuristics, analogies and strategies available at the time, etc.
Fourth step	To analyze and compare solutions • Studying those that occurred historically and compare them with the solutions in the current context.

An example: logarithms. These can be placed in the context of navigation on the high seas (15th century, from the conquest of America) that promoted astronomical study, since it required extensive numerical calculations and the use of arithmetic and geometric series, from which the idea of logarithm. You can study the way Briggs, Napier and Bürgi worked, or you can go to logarithmic tables and the use of software for consideration. Passages from old books containing the tables can be used.

Another example: constructions with a straightedge and compass. This would make it possible to study the context of ancient Greece (such as the presence of prejudices about the irrational), to use the instruments for a specific test, to present the matter in modern terms and with other resources, and to contrast the different conceptions.

The use of successive subtractions can also be cited to calculate the common divider of two numbers when applied to the side and diagonal of a regular pentagon. It turns out that since these are not commensurable, the process continues indefinitely and in this way an opportunity is given to study one of the most famous irrationals: the golden ratio.

V. ASSESSMENT

This chapter summarizes principles and general guidelines for the development of an effective assessment in the teaching of Mathematics. In the same way, the percentages for the components are obtained and indications are presented for some of them.

Assessment of learnings

Assessment as an integral part of the teaching and learning process has the purpose of collecting valid and reliable information that allows determining to what extent the capacities or competence proposed in the curriculum are achieved.

In this sense, teaching action is facilitated in making prompt and timely decisions aimed at improving student performance. In the same way, it provides information about this performance, conceptual understanding and application, and reflection on problem solving.

From this perspective, assessment should not be seen as an isolated activity with a punitive sense, but as a process inherent to pedagogical mediation, which allows each student to build learning from their experiences, thus transcending the idea of assessment as a sanction mechanism.

The Costa Rican Mathematics curriculum has problem solving as its main focus as a methodological strategy. This implies a change in the pedagogical mediation processes, starting from an organization of the lesson, where the introduction and learning of new knowledge is promoted, following four steps o central moments: the proposal of a problem, independent work, interactive and communicative discussion and closure. This leads to a change in the assessment process, which begins with the rethinking of the educational task and the way in which educational activities are planned, developed, and evaluated.

If the focus is kept on knowledge retention, the assessment will continue to be a process to request rote responses and the mechanical resolution of exercises. A situation that is contrary to the approach of this program.

Principles

To approach this curriculum, it is important to **plan** the assessment of learning based on the following principles:

- ***It is an integral part of the teaching and learning process.*** Assessment should not be considered a separate process from pedagogical mediation, or as a set of tests applied at the end of a unit or topic. It must become a natural part of the learning process, which takes place during the activities that are proposed in the class, when each student participates, listens, analyzes situations in the environment and proposes strategies for their solution considering the different levels of complexity.

- ***It is a collaborative process.*** Each student learns from his classmates and from the teacher, and the teacher learns from and with her students. The formulation of activities that involve putting into practice the abilities and competencies of students favors the development of their self-esteem, values, and attitudes, as well as the promotion of positive beliefs regarding the subject.

- ***Pertinence with mediation activities.*** During the development of mediation activities, it is necessary to collect qualitative and quantitative information about student performance in the different mathematical areas. The information collected through technically elaborated instruments will allow you to evaluate abilities and competencies, and decision making.

- ***Congruence of techniques and instruments.*** The techniques and instruments used in the assessment process must be varied and appropriate to the level that it intends to evaluate. They must serve to reflect the level of knowledge and the specific skills achieved.

- ***Allows decision making.*** The analysis of the information collected allows reflection on the pedagogical practice and decision-making aimed at feedback or reorientation of it. In addition, it allows identifying the strengths in the learning of each student and suggests how to develop them even more, with clarity and a constructive attitude regarding possible weaknesses and the ways in which they could face them.

  For each student, for his part, it allows reflection on performance and self-assessment, so that over time there is responsibility for individual learning, and request, if necessary, for the support that facilitates the development of the capacities proposed in this curriculum.

- ***Promotes commitment to learning.*** To guarantee effective learning, there must be student understanding of what the learning objectives are and the desire to carry them out. This understanding and commitment to the students own learning comes when they are aware of the goals and criteria that will be used to assess their progress. Clear communication of these criteria involves formulating them in a way that understands what is expected of student performance.

Assessment in problem solving

The assessment must be registered within situations that carry meaning and that cause cognitive imbalance, from which the development of new abilities and student capacities is favored.

A problem is organized around an obstacle to be overcome that has been previously identified and planned. This situation must represent a challenge that provokes the student effort to respond to it, putting into practice knowledge, capacities, and competence.

When considering a problem as part of the assessment of learning, not only the results should be identified and assessed, since their meaning would be lost, it is also appropriate to consider the following phases:

- The exploration of the problem.
- The establishment of the strategy.
- The development of the strategy.
- Self-reflection on strategy.
- The analysis of the results.
- The conclusion.

It should be considered that the learning of Mathematics is progressive to the extent that it is developed based on the achievement of knowledge that is fundamental for the achievement of other more complex knowledge. And it is operational, insofar as knowledge of the concepts is not enough, but rather their application in solving mathematical problems with different levels of complexity.

When dealing with different levels of complexity in the mathematical problems to be developed in the classroom, it is appropriate to identify the possible actions of each group of students. Not all problems raised can be evaluated in the same way. If it's a playback issue, it's easier to measure performance in terms of results. However, if it is a “reflection” problem, this is not enough. A “middle ground” can be “connection” problems.

When it comes to more complex problems, for example of connection and reflection, it is necessary to diversify the instruments and assessment techniques, with the purpose of obtaining information about the mathematical processes developed, the products obtained, with the purpose of providing support in the development of skills and competencies.

Assessment strategies must be consistent with the level of complexity of the problems. In the case of the summative assessment, it would be totally wrong, for example, to design a test in which the problems of the reflection group predominate. For this reason, for its design the balance between the different levels of the problems included must be guaranteed, considering for this the approach carried out during the process of pedagogical mediation.